GALILEO RISING

THE PITTSBURGH PLAN
HANDBOOK FOR PRESCHOOLER INTELLIGENCE

THE PITTSBURGH PLAN

The Pittsburgh Plan

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FOREWORD

This book describes the Pittsburgh Plan, an exciting new program for helping young children learn to read and do mathematics – and, in the process, to grow intellectually, develop confidence, and enjoy the thrill of learning.

We Pittsburgh Planners believe that your child will learn best by doing. In that spirit, we encourage you to jump right into the Pittsburgh Plan. Get started right now, and read the book as you go! Here is how you can get started today, within a few minutes:

1. <u>Download Appendices A and B.</u> If you have not previously done so, download Appendices A and B from our website, www.pittsburghplan.com. Appendix A is a complete set of "Sample Homeworks" and Appendix B is our Quickstart Module.

2. <u>For Most Very Young Children</u>: Look at the worksheets in the Quickstart Module. If they are suitable for your child (i.e., not too easy), just read the Quickstart Module and follow its instructions.

Note that we do quite a bit even with simple worksheets, so if you are not sure about your child's appropriate starting point, start at the beginning (Sample Homework #1 in the Quickstart Module) and see if he or she is interested in what you are doing.

Once you have begun sessions under the Quickstart Module, start reading this book.

3. <u>Fundamental Rule</u>: Keep It Interesting. If a Sample Homework worksheet contains material that is too easy for your child and that your child finds uninteresting, skip that material; move forward to something interesting.

4. <u>For Advanced Older Children:</u>

(a) <u>Determine where in the materials your child should start.</u> You can do this by flipping through the Sample Homeworks of Appendix A. The chart in the section entitled "Later Entry Considerations" may help. Pick the first Sample Homework that contains significant material that might be new or interesting to your child. (And remember rule 3 above!)

(b) <u>General Background.</u> Read the Quickstart Module for general background on the Pittsburgh Plan's methods and approaches.

(c) <u>Detailed Instructions.</u> Detailed instructions for each Sample Homework Sheet are contained in Chapters 10 – 13 below. Find the Chapter that covers your starting-point Sample Homework and read that Chapter up to and including the specific instructions for your starting point Sample Homework.

(d) <u>Get Started.</u> Then get started on your first "homework session" with your child, and read this book as time permits.

INTRODUCTION AND FAQ

With apologies to Dr. Spock: Trust your child. She can do more than you think she can.

In 1946, Dr. Spock's The Common Sense Book of Baby and Child Care created a revolution in child care by teaching parents how to take charge of their children's health and physical development. Reviewers scoffed; after all, how likely was it that American parents would want to be responsible for their children's health? Sixty years later, we can scoff at the reviewers; it is clear that Dr. Spock was addressing a very real need, the need of parents to help their children be healthy and thrive.

Today, American parents are desperate for a different kind of help, namely, help on the intellectual side of child development. They are desperate to help their children get good grades, do well on the SAT, enter good colleges, and compete effectively in the job market for rewarding careers.

Parents are trying everything, and nothing is working. Some families pay tens of thousands of dollars each year to send their children to exclusive prep schools – including exclusive preschools! – just to give them an edge in this grim competition. More than a million American families have resorted to the even more extreme measure of home-schooling, taking on the huge burden of their children's entire education rather than trusting the schools to do their job. This desperate focus on the intellectual development of our children is not a passing fad, it is a national imperative. In fact, it is the direct cause of the famous (or infamous) "No Child Left Behind" legislation, now known colloquially as "No Child Left Untested."

The situation is indeed grim, but do not despair; there is another path, a less desperate path. Home schooling is not necessary; elite private schools are not necessary; all that is required is that you spend a small amount of time with your child in the manner described in this book.

What does this book do? It outlines a simple program – the "Pittsburgh Plan" -- for helping your pre-school child reach her full intellectual potential.[1] It is not a home schooling book; it does not ask you to replace your child's teachers. Instead, like Dr. Spock's famous book, it is a parenting book. It helps you do better the very things that you will try to do for your child in any event; it helps you teach your child to read, to understand math, to think with power and clarity. And it is not burdensome; all it asks is that you spend a few minutes each week with your child.

The results are almost unbelievable. Children who begin the program at ages two to four routinely enter elementary school years ahead of grade level in both math and reading, and this pattern of excellence and acceleration continues throughout school – e.g., AP calculus in tenth or eighth or even sixth grade, SAT scores in the very high 1500's (old SAT) and very high 2300's (new SAT), extraordinarily high grades and class ranks in high school, and continued top-level performance in college courses.

1 We ask your indulgence concerning our use of the feminine generic forms – "she" and "her" – when referring to children. This practice is intended to honor the extraordinary achievements of our female participants.

Case Study #1: *"Elmer" began the Pittsburgh Plan at age 2½. By age 6 he had read the first two Harry Potter books and was able to do very advanced math work, including solving single-variable algebra problems such as ½(4x-3) = 3+2x/5, expressing 3/7 as a decimal, doing simple trigonometric work (sines, cosines, etc.) with angles in both degree and radian measure, solving word problems involving rates – and much, much more. Elmer was the top student in his AP calculus (BC) class in seventh grade and achieved the top score in Pennsylvania (and one of the top few scores in the nation) among the thousands of gifted 7th graders taking the SAT test with a score of 760 verbal, 760 math, and 740 essay. Now in eighth grade, Elmer is taking multivariate calculus at the University of Pittsburgh.*

How are such results possible? Quite simply, they are the result of the Pittsburgh Plan's novel approach to the learning of reading and mathematics. Everyone knows that young children have an incredible ability to learn to speak a language, but until now no one has attempted to enlist this powerful ability to aid in the learning of reading and, especially, mathematics.

The Plan does exactly that. Drawing upon the author's strong background in psycholinguistics, child language acquisition and mathematics, and upon sixteen years of intensive work with scores of young children in developing approaches and materials, the Plan allows any parent, regardless of background, to help a child learn mathematics and reading in the same way that the child learns to speak a language. As a result, the child learns to read and to do mathematics effortlessly and well – and in the process, the child becomes much more intelligent, because these new skills are learned in the same deep way that a native language is learned.

Case Study #2: *"Valerie" began the Pittsburgh Plan at age 3½. By age 6 she was able to read chapter books such as Boxcar Children, and her level of math achievement was comparable to that of Elmer above. Valerie won top-student honors in Honors Calculus in eighth grade, and then took and excelled in Calculus 3 (while in ninth grade) and Introduction to Math Theory (while in tenth grade) at the University of Pittsburgh. She went on to qualify as the top student in advanced Real Analysis and Abstract Algebra courses at Carnegie Mellon University while in 11th and 12th grades, respectively. Valerie achieved a score of 2360 on the new SAT on her first try, was a National Merit Finalist, was accepted at numerous elite colleges (e.g., MIT), and is attending a large public university on a full scholarship. Because Valerie started elementary school a year early – not advised!! – she entered college at the age of 17.*

And this is not all. As the child – your child! -- experiences the joys of rapid intellectual breakthroughs and the mastering of difficult material, she will come to love learning, to thrill in the exercise of her intellectual powers in the same way that an elite athlete glories in the exercise of physical powers. And she will develop great self-confidence, a strong and justified sense of self-esteem.

Case Study #3: *"Rachel" began the Pittsburgh Plan at age 2. She was able to read chapter books before reaching age 4 and by age 6 she was able to do very advanced math work, similar to Valerie. Rachel achieved an "A" in AP Calculus in sixth grade, and then (while in seventh grade) was the top student in her Calculus 3 class at the University of Pittsburgh, and (while in eighth grade) was the top student in her math theory course at Carnegie Mellon University. As a seventh grader, Rachel achieved the top score in Pennsylvania on the SAT (amongst the thousands of seventh graders who took the SAT that year); as an eleventh-grader, she achieved a perfect PSAT score of 2400 and an SAT score of 2360 and is now attending Yale University.*

In short, by making a small investment in your child, an investment not of money, but of time, of just an hour or two a week, you will give your child a giant boost in life: she will develop a passion for learning and for thinking; she will become confident and assured; and she will succeed academically far beyond your expectations.

Case Study #4: *"Elise" began the Pittsburgh Plan at age 4. By age 6 she was able to read chapter books such as Charlotte's Web and do math problems such as 43x87, 289+896, 1,012-687, 289÷4, as well as adding, subtracting, multiplying and dividing fractions and decimals – and much, much more. Elise went on to win top-student honors in AP Calculus in tenth grade, achieved a score of 1580 on the old SAT on her first try, ranked first or second in her class of over 800 throughout her entire high school career, was admitted into numerous elite colleges, and is now attending Harvard Law School. Elise had her first short story published in an anthology of science fiction stories when she was sixteen.*

How do we achieve these results? The approach is simple. We treat the brief "homework" sessions as play time between the parent and child. There is never any pressure; there is simply happy, sit-on-the-lap time that focuses on carefully selected academic topics. Of course, although you know the topics are academic, your child does not. She thinks you are playing.

Always honoring this fundamental principle of "playtime – no pressure," we begin teaching the elements of reading and math at a very early age – as early as twenty-two months, and usually by the age of four. (Chapter 10 discusses how to tell when your child is ready to enter the Plan.) Although such early starting ages are preferred, children may begin the Plan at ages of five, six or even older and still benefit substantially. In such "later entry" cases, the earliest parts of the Plan can be omitted or presented very rapidly until material that is new to the child is reached. Such "later entry" cases are discussed below in this chapter, immediately after the "Frequently Asked Questions," under the heading "Later Entry Considerations."

We focus at first primarily upon reading, until we have reached the point where your child reads by herself for fun. During this phase, and at all times thereafter, we urge parents to spend a great deal of time reading with their children, just as they would have done absent the Pittsburgh Plan. In other words, the Pittsburgh Plan is asking for a small amount of EXTRA time with your child each week; it is not intended to replace normal parent-child time.

Once a child has achieved sufficient reading skill to read by herself, it has been our experience that she becomes an avid reader – that reading takes on a life of its own for her, and no longer requires as much attention in the homework sessions. At this point, we turn our primary focus to the study of math. Why emphasize math? After all, math is not as important as reading for most people. The answer is simple; the standard expectations for childhood math achievement are so low that they provide us with a huge opportunity, the kind of opportunity that businessmen refer to as "low-hanging fruit."

It is easy to outperform the math objectives of the elementary school curriculum. It is easy for most children to become proficient at math when measured against their peers and against the standard curriculum. So this is an opportunity to allow our children to experience the joys of academic success and develop self esteem that will endure, self-esteem that is in fact earned and deserved.

There is another reason why we focus on math. The principal theory underlying the Pittsburgh Plan is that math can be learned as a native language. This theory strongly supports the early introduction of advanced math topics, and suggests a number of approaches to math that are very different from the standard American approach. The fundamental innovations of the Pittsburgh Plan lie in the area of math education. We will discuss this in more detail in later chapters.

What happens if you put a child through the Pittsburgh Plan? As illustrated by the above Case Studies, children who begin the plan before the age of four often read early and become voracious readers of chapter books by second grade. Gifted children will read earlier, sometimes much earlier. It is not unusual for children on the Plan to read books such as the Harry Potter books in kindergarten or first grade. Please note that reading success under the Plan depends heavily on parental support activities, especially frequent reading to and with the child, and also on creation of a family environment where everyone – parents, siblings, pets – reads for fun.[2]

The uniqueness of the Pittsburgh Plan lies in the area of developing math skills in children. Based on the theory discussed above, the Plan adopts an unusual language-oriented, problem-based approach to math education. The results have been impressive. What can you expect if your child begins the Pittsburgh Plan at an early age? Our experience suggests that your child will have mastered the following (among many other things) by the beginning of first grade:

- Recognition and identification of up to seven-digit numbers;
- Multi-digit addition including carrying (e.g., 689 + 974);
- Multi-digit subtraction including borrowing (e.g., 2,012 – 988);
- Two-digit multiplication (e.g., 12 x 31);
- Multiplication with carrying (e.g., 29 x 3);
- Simple division;

2 Just kidding about the pets.

- Recognition and identification of fractions;
- Geometrical meaning of fractions;
- Simple addition of fractions (e.g., 2/7 + 4/7); and
- The place system for base 10.

Think about this for a minute. What do you think this kind of performance does for a child's self-image and confidence? Remember, the standard list of first-grade math objectives includes number recognition to 100 and rote memorization of basic addition and subtraction facts, facts such as 2 + 4 = 6. A child who has been through the Pittsburgh Plan soars above these expectations. She knows - knows, not hopes or wishes - that she is able and competent. She is an excellent reader and thinker, and she has no fear at all of math; in fact, she loves math, because she is so good at it.

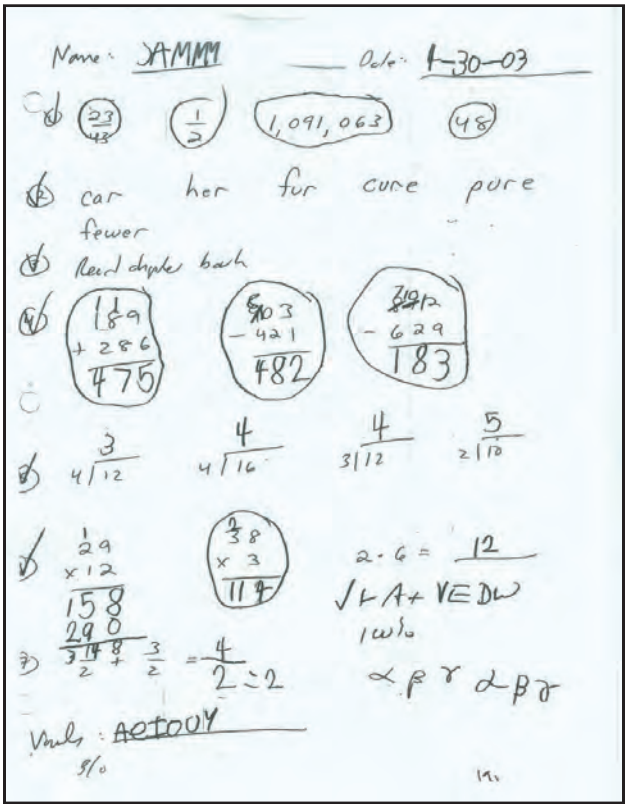

FIGURE A: This figure shows an actual homework sheet (the same as Sample Homework #191 of Appendix A) that was done by a Plan participant who had just turned six and was six months away from entering first grade, The work shown here is fully representative of the average level of work done by Plan participants.

Is this really possible? Or are we exaggerating just a bit? Well, take a quick look at the homework sheet in Figure A. In an eighteen-minute session, Sammy, who was still in kindergarten, recognized and correctly read "23/43" and "1,091,063," read three pages from a chapter book (in this case, Charlotte's Web), correctly worked "189+286," "903-421" and "812-629," did four simple division problems, correctly multiplied "29x12" and "38x3," and added "1/2 + 3/2" to get "4/2," which she then correctly simplified to "2." Sammy did all of these problems quickly, without strain, in a very matter-of-fact way; in other words, these were not difficult problems for her, they were business as usual! And this type of work is not just business as usual for Sammy; it is precisely the level and quality of work that we see routinely from young Plan participants.

As illustrated by the above Case Studies, this pattern of mathematics achievement and confidence continues into later grades. For example, children participating in the Plan successfully complete high school courses in AP Calculus as early as sixth grade and often by tenth grade; they do extremely well on standardized math tests (such as the mathematics portion of the SAT); and they go on to do extraordinarily well in college math courses, often while still in high school.

Unlike many approaches to child learning and self-esteem, the Pittsburgh Plan is based on a careful theoretical framework, the key elements of which are explained in the following chapters. In essence, that framework boils down to one very important idea: we trust your child to learn, rather than trusting you to teach. Why? Because – and there is really no way to say this gently – your child is almost certainly much smarter than you! (That is why we call this book "Galileo Rising" rather than "Newton Teaching.") In particular, your child has an amazing ability to learn languages inductively, simply by being immersed in them. Under our program, like a weary hobo hopping onto a high-speed train, math and reading simply hitch a ride on your child's magnificent language-learning ability.

Does the Pittsburgh Plan work for every child? No, it does not; in truth, there is no system that suits every child. Many families try the Plan and then abandon it. But a significant percentage of the children who have tried the Plan have enjoyed it and stuck with it, and in their cases the results have been extraordinary, as described above. And in cases where the Plan does not work, there is no harm done!

Look at it this way: the Pittsburgh Plan is a flexible and powerful tool that is available to you to help you achieve your goals as a parent. Use it if you find it to be helpful; otherwise, store it with the exercise bicycle and the diet books that you have every intention of using someday.

Let me take just a minute here to say something to you, parent to parent. In a way, it is prompted by the preceding two paragraphs, where I point out that it is possible that the Plan might not work for you.

When I completed the first draft of this book several years ago, a well-known literary agent advised me to eliminate this sort of "negative admission" because it might discourage people from buying the book. I found myself rebelling against this advice. You see, from my perspective, these were not "admissions," they were just statements. In a way, it would be dishonest not to say these things, because otherwise parents who tried the Plan without much success might feel that they had somehow failed in a project where everyone else was succeeding!

". . . like a weary hobo hopping onto a high-speed train, math and reading simply hitch a ride on your child's magnificent language-learning ability."

And as I thought further about this, I realized that I have a heavy burden in this book, because I am making what appear to be outlandish claims. I am claiming that young children can achieve things that are almost unbelievable! After many years of experience, I am well aware that the initial reaction of teachers, educational psychologists, and other experts to these claims is one of polite disbelief. They do not doubt my motives; they just are quite certain that children are not capable of achieving at these levels.

All that I have ever been able to offer in support of my claims are results. And, over the years, it seems that the results have been enough; the Pittsburgh Plan has gradually expanded as parents, teachers and educators have seen its effects. It has spread by word of mouth, almost subversively, as parents have given copies of the book (in loose-page, pre-publication format!) to their friends and relatives. And gradually it has begun to win acceptance in academic and educational circles, possibly due to the growing number of university professors who are using the Plan with their own children.

I have come to realize that my negative reaction to the advice of the literary agent sprang in part from my long experience of having the facts as my only allies against the presumption that children simply cannot do what the Plan claims they can do. For years I have been very careful to hew to the facts; I am not about to change that approach now.

So here is my pledge to you: To the best of my knowledge, every single statement of fact in this book is true. If I claim a result, it exists and I can support it. And if I am aware of an issue or limitation, I will tell you about it.

One more point: You know these extraordinary claims that we are making? They are not claims about the Pittsburgh Plan; they are claims about the capabilities of your child! Over and over again, you will read in this book that our main objective is to get out of the way of the child and allow her to reach her full potential, unhindered by our preconceptions. The Pittsburgh Plan is a merely a tool, one possible means to an end; the star of the show is your child.

The book is organized as follows:

Part 1 (Chapters 1 – 5) describes the origin of the Pittsburgh Plan and argues for the need to do something radically different for our children in the area of early math education. In particular, these chapters discuss the need for a new approach to mathematics education for young children, and then describe the basic theoretical underpinnings of the Plan. These chapters explain why the homework materials are structured as they are and provide context for the detailed discussions of those materials in the ensuing chapters.

Part 2 (Chapters 6 - 8) outlines the overall approach a parent should take to the brief homework sessions, and describes key objectives and important guidelines for the parent's role in those sessions.

Parts 3 and 4 (Chapters 9 - 13) lay out a complete description of the Pittsburgh Plan, providing many examples of homework materials and a detailed day-by day discussion of the entire program from start to finish. These chapters are the nuts and bolts of the book, and provide the reader with everything that is needed to take his or her child from the beginning to the end of the Plan in a manner that can easily be customized and tailored even by the non-expert parent. Frequent references are made to the materials in Appendix A.

Part 5 (Chapter 14) discusses next steps after completion of the Pittsburgh Plan.

Appendix A is our "canned Plan" -- an actual set of homework materials that were used with a participant in the Pittsburgh Plan. These materials can also be downloaded from the Plan's website, www.pittsburghplan.com, so that they can be printed out and used more than once by the reader (and also can be modified as needed).

Appendix B is a Quickstart Module that provides readers with everything they need to begin the Plan right away, before reading another word of this book. In other words, as you read this sentence, you are less than a half-hour away from beginning the program that could change your child's life! So go ahead -- get started right away using the Quickstart Module (Appendix B) and read the rest of the book as you go! (You may wish to download Appendix B from the Plan's web site, www.pittsburghplan.com – note that all downloads from our website are free.)

There are two ways to launch the Pittsburgh Plan with your child. If you want to get started immediately, you can use the Quickstart Module (Appendix B). That module is easy to use -- self-explanatory and self-contained. Alternatively, you can enter the program more deliberately by reading Chapters 6 – 9 and then beginning the homework sessions as described in Chapter 10. Even if you elect to use the Quickstart Module, it would be worth your while – although it is not required! – to read these Chapters.

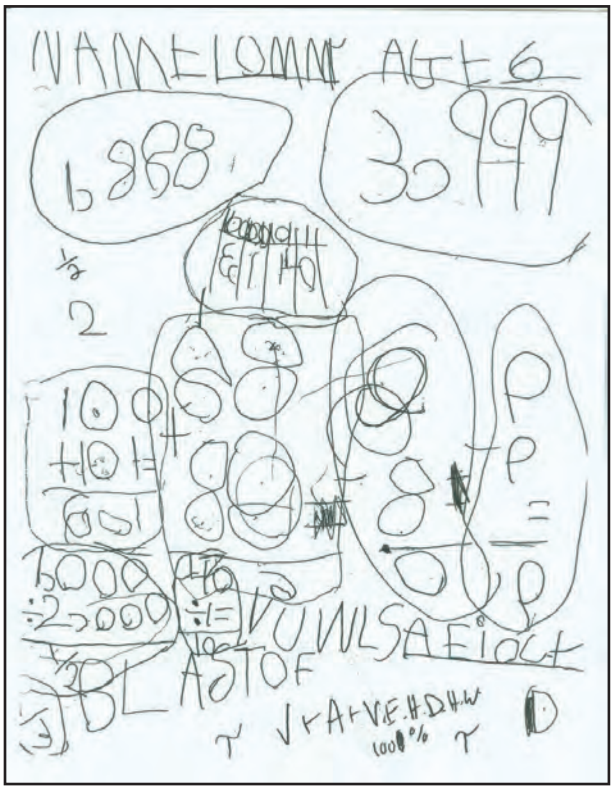

FIGURE B: A homework sheet that was created spon-taneously by a six-year-old Plan participant

After you have begun the Plan by using one of these two basic approaches, you have a great deal of flexibility in how you continue the Plan. At one extreme, you can simply use the materials in Appendix A as a "canned Plan" in the manner suggested by Chapters 9 - 13. Alternatively, you can use Appendix A and Chapters 9 - 13 as a template and design your own homework materials. In either case, it is important that you read Chapters 6 – 8 carefully in order to understand the basic ideas behind the Plan.

In our experience, almost everyone who tries the Pittsburgh Plan starts by using Appendix A as a canned plan, but it is quite common for parents eventually to switch over to the more customized approach. Why? Because the homework sessions are fun; parents end up being captivated by their children's enthusiasm and proficiency. In fact, you may find that even if you do not create customized homework materials, your child will! You will know that things are going well when your child approaches you with a homework sheet that she has prepared herself and asks you to do it with her.

Figure B above shows an example of a child-generated homework sheet. Be warned that it is common for Plan participants to do this on "off" days; be happy when it occurs, because there is no surer sign that you are succeeding in the primary objective of keeping the homework sessions pressure-free. At first glance, you might be put off by the messiness of this paper and the fact that the child occasionally writes his numbers backward. Far more significant is the fact that the child, on his own, generated and solved difficult problems such as 1,000÷2000 = ½, 10÷10 = 1, and 88+88 = 176, and wrote out a base 10 place system chart up to the thousands column!

Frequently Asked Questions (FAQ): Certain questions seem to come up repeatedly when the Pittsburgh Plan is presented to parents. You may find it helpful to browse through the following list of questions, along with our carefully thought-out responses:

Q: Will the Pittsburgh Plan increase my child's IQ?
A: Probably.

Q: Will the Pittsburgh Plan increase my child's SAT score?
A: Probably.

Q: How will this affect my child's chances for a career in professional sports?
A: There is no effect at all. Your child's chances are essentially zero with or without the Plan.

Q: Why do you call it the "Pittsburgh Plan?"
A: Some people name their plans after glamorous locales for no reason other than to capitalize upon our national fascination with the rich and famous. In a sense, they are trying to sell books under false pretenses. After all, what exactly does the "South Beach Diet" have to do with South Beach? Other people name their plans after themselves (the "Pritikin Diet," etc.), possibly for reasons having to do with ego. On those theories, we could have named our plan the "Beverly Hills System." But our objectives are a little different. We want to help children become smart!

What does all of this have to do with Pittsburgh? Pittsburgh is a craggy city, lacking in sophistication and glamour, but it is also a city of strong families and neighborhoods, populated by hard-working parents who will do almost anything to help their children have a shot at a better life. If you want a glamorous plan, if you want to spend money rather than time on your child, if you want your child's education to reflect your social status and privilege, then you should look elsewhere. If you just want results and are willing to work for them, then you are at heart a Pittsburgher and you have come to the right place. You have come to the "Pittsburgh Plan."

Q: I am very busy. I just don't have time to do the Pittsburgh Plan.
A: That is not a question.

Q: How can a professional, working parent find time to do this Plan?
A: Get up fifteen minutes earlier, or skip the first segment of "Wheel of Fortune" four times a week.

Q: My child is very busy, and I don't see how I can ask her to add yet another activity to an already crowded schedule.
A: This is not an activity, it is time with a parent.

Q: Well, whatever. It is still a problem, given my child's schedule. What do I do?
A: Reduce your child's TV time to 3 hours and 45 minutes a day (from the national average of four hours a day), or forgo gymnastics or soccer or tae-kwon-do or piano or Celtic Dance or acting or tee-ball or basketball or . . . . you get the idea.

Q: But won't that put my child at a competitive disadvantage vis-à-vis the other kids who are receiving training in those activities?
A: I'll stick my neck out and hazard a guess that your child's competitive advantage at reading and math will outweigh any competitive disadvantage at Celtic Dance.

Q: Why should I trust this program?
A: You should trust it because you are administering it. You control every aspect of it. Worst case, if everything in this book is wrong, you are spending an hour a week with your child. This is not exactly a dangerous downside scenario!

Q: How does your plan compare with the other programs for young children that are out there – programs such as those offered by Junior Kumon and Score!?
A: We share with those programs a belief that young children are far more capable than is usually imagined, and that we should not erect artificial barriers (through our limited expectations) to their intellectual progress and growth. That said, we differ from those programs in many ways. Most importantly, (i) the Pittsburgh Plan is not <u>our</u> program, it is <u>your</u> program; we merely provide materials that help you work with your child, (ii) let me say that again, in another way, because it is so important – our program does not provide lessons taught by a paid professional instructor, it provides you with materials and asks you to share time with your child, one-on-one, (iii) we are committed to avoiding putting pressure on the child, and have structured our program with that as our highest priority, (iv) our program emphasizes child-driven learning rather than adult-driven teaching, with materials and approaches that are carefully structured to enlist the child's innate inductive learning ability, (v) as a result we help a child build intelligence rather than rote knowledge, and (vi) those other programs cost many hundreds or even thousands of dollars; the entire cost of our program is the cost of this book, a little paper, and a few pencils – oh, and an hour a week of your time and attention.

Q: Why are your drawings and other graphics so crummy?

A: At the most basic level, the answer is that we are not trying to impress you; if that were our goal, we would call our plan the "Beverly Hills Plan." Instead, we are trying to reach your child, and it turns out that the crummy graphics are an important tool for achieving this goal. Certainly, it would be easy enough to make the Sample Homeworks highly polished and professional (and in fact we have done that on occasion), but this actually tends to separate the child from the process. Part of the charm and power of the homework sessions is that they are special times between a parent and a child, not just the presentation of one more mass-produced book or toy, but an organic process that is at its very core a deep collaboration between parent and child. Our graphics – in fact all aspects of our presentation – are intended to be within the reach of your child, allowing her to feel that she is an active participant with you in the homework sessions.

Q: What if my . . .

A: Let me add two more notes about our crummy graphics. First, the graphics in the Sample Homeworks were created (with a bit of guidance) by children who had recently completed the Plan. Second, it will probably enhance the homework sessions if you use the Sample Homeworks as guides, but actually copy them over in your own handwriting, modifying them as you see fit to tailor them to your child. She will treasure your active involvement. (Do not think for a minute that she does not notice such things!)

Q: What if my child feels pressure?

A: Although one might suspect theoretically that this would be a possibility, in practice it simply has not been a problem. That said, remember that the first, most important rule of the Pittsburgh Plan is to keep your sessions free of pressure. You know your child better than anyone, and are well-equipped to accomplish this. If ever you sense that your child is feeling pressure, back off immediately and resume another day, possibly with material the child has already mastered. In the end, if the child feels pressure no matter what you do, just stop the program and do something else with your child.

Of course, you should use your normal parenting skills to help assure that sessions occur with reasonable regularity. For example, do not ask, "Would you like to do a Homework now, Roscoe?" Every parent in the world knows that Roscoe's default answer to such questions is "NO!" Instead, try something more effective from your typical bag of parent tricks, e.g., "Roscoe, it is time to do your Homework! Let's hurry, before the letters run off of the page!" Or, "Roscoe, let's do your Homework now – or would you rather clean up your room first?"

Q: I am one of those people who is terrible at math. I could never teach it to my child. How can I do the Plan?

A: This book presents the Plan in several different ways, designed to fit the needs of different parents. Perhaps you should begin by using the canned Plan in Appendix A. Once you get going, you will realize that you know plenty of math to handle the more flexible approach (after all, we are teaching very simple concepts). The Plan is designed with parents like you in mind, and you will most likely find that it works very well with your child. But, whether or not you have any math ability at all, your active, enthusiastic involvement is critical to the success of the Plan. This is where you can match any parent, anywhere – bring your love of your child, and your joy at spending time with her, to the homework sessions; the Plan will handle the rest.

Q: Can you recommend any resources for me and my child?

A: You will find an updated list of recommended resources on our website: **www.pittsburghplan.com** (under the "Resources" tab).

LATER ENTRY CONSIDERATIONS

Much of the Plan's success derives from the sense of achievement that participants feel as they master new skills at an amazing rate. This sense of being on the cutting edge, of being an intellectual explorer, can be undermined if the child's first exposure to the Plan involves material that he or she already knows -- a particular risk in the case of entry into the Plan by children who are age 4 or older.

What should you do if the earliest Sample Homeworks look too easy for your child? You probably can guess the answer: start your child at a later Sample Homework! Flip through the Sample Homeworks until you reach a level that you think is about right for your child as an entry point, and begin there. Remember also that you do not have to do the entire Sample Homework – feel free to work only on the parts that will be interesting to your child. There is nothing wrong with a short but interesting homework session; there is a lot that is wrong with a dull homework session.

The following chart describes where in the Sample Homeworks key skills are introduced, and may help you determine the appropriate entry point for your child:

Main Skill	Component Skill	Introduced In:	Key Sample Homeworks:
Addition	Basic	SH #34	
	Advanced	SH #68	
	Two-Digit	SH #106	SH #124
	Carrying	SH #131	SH ## 132, 138
Subtraction	Backward Counting	SH #23	
	Basic	SH #52	
	Advanced	SH #98	
	Two-Digit	SH #124	
	Borrowing	SH #146	
Zero		SH #50	SH ##52, 55, and 62
Multiplication	Basic	SH# 143	SH #150
	Identity, Zero	SH #161	
	2-Digit x 1-Digit	SH #167	
	With Carrying	SH #170	SH #182
	2-Digit x 2-Digit	SH #189	
Division	Basic	SH #183	SH #185
Place System	Basic	SH #116	SH ##121, 123 and 126
Fractions	Basic	SH #67	SH ##71, 123
	Adding	SH #179	SH #180
Dimensions	Basic	SH #162	SH #162A
Sounding Out Words	Basic	SH #35	
	Advanced	SH #67	SH #73
	Two-Vowel Rule	SH #73	SH #78
	Silent "e" Rule	SH #74	SH #78
Reading Sentences		SH #64	

At first, the Pittsburgh Plan is primarily a program for teaching early reading skills to young children. However, the focus of the Plan shifts relatively quickly to a heavy (although not exclusive) emphasis on math. This is the heart of the Plan; the place where it appears to make a real difference in the lives of its participants.

The following chapters discuss the origin of the Pittsburgh Plan as a solution to four interrelated plagues of modern American math performance: (i) the gender gap; (ii) the class chasm between rich and poor children; (iii) the performance gulf between American children and international children; and (iv) the pervasive failure of American society to give children the math experiences and education that they deserve. A fifth plague, the vast math/science divide between racial and ethnic groups in America, is not addressed directly by the Plan, but is addressed indirectly insofar as it is correlated with the four problems listed above.

These chapters serve two purposes. First, they are motivational; they provide strong incentives to try the Pittsburgh Plan and to stick with it. Second, by presenting some of the thought processes behind the development of the Plan, they help the reader understand the objectives and methods of the Plan. In a sense, these chapters are the equivalent of the first few chapters in most diet books, where the dread consequences of high cholesterol, obesity, and a sedentary lifestyle are laid out in gruesome detail. In our case, the gruesome picture is of a self-perpetuating culture of math anxiety and under-performance.

As in the case of the diet books, these motivational chapters are not strictly necessary, and the reader who wishes to get started immediately on the Plan should feel free to skip directly to Part 2.

Chapter 1: *Bridging the Gender Gap*

"[U]nfortunately the passion of the moment predominates, in the Frail Sex, over every other consideration. This is, of course, a necessity arising from their unfortunate conformation. For … they are … wholly devoid of brainpower, and have neither reflection, judgment nor forethought, and hardly any memory."

--- **Edwin A. Abbott,** Flatland: A Romance of Many Dimensions, 1884

"Indeed, Flatland has … been 'read by every self-respecting physicist, mathematician and science-fiction writer . . . .'"

--- **Peter Renz,** Scientific American, April 2002

"For various biological and hormonal reasons, boys seem more suited than girls to deductive reasoning and abstract thought."

--- **Cynthia Brantley Johnson,** "What Smart Girls Know About the SAT,"
How to Get Into College, Kaplan/Newsweek, 2003

"There are three broad hypotheses about the sources of the very substantial disparities . . . with respect to the presence of women in high-end scientific professions . . . the first is what I call the high-powered job hypothesis [i.e., that women do not want high-powered, intense jobs]. The second is what I would call different availability of aptitude at the high end, and the third is what I would call different socialization and patterns of discrimination . . . .And in my own view, their importance probably ranks in exactly the order that I just described."

--- **Lawrence Summers,** then-President of Harvard University,
in oral remarks made on January 14, 2005.

"Math is hard! I love shopping!"

--- **Teen Talk Barbie**, 1992

You have a young daughter. You would like her to do well in school, go to a good college, and have a satisfying and fulfilling life. You do not care if she chooses to be a housewife or a career woman so long as the choice is not forced upon her. Although our society still stereotypes girls and women, you believe that you can help your daughter overcome the obstacles created by such stereotypes. But, you are troubled by the pervasive reports that post-pubescent girls do worse than their male counterparts in almost every measure of math achievement. Put simply, girls do just fine in math until they reach middle school, but thereafter lag behind boys in many measures of math performance.[1]

What can you do about this problem? How can you help your daughter close the gender gap? In 1991, I faced this issue squarely as the father of three young girls. First, I researched the question, only to find that there is no real consensus as to why girls have such difficulties with math. Some authorities suggest that it is a constitutional phenomenon – that girls are less aggressive than boys and thus cannot compete in the rough and tumble of a middle school classroom. Others suggest that it is a result of stereotyping - that girls fall into expected behavior patterns, including a "feminine" (i.e., unimpressive) performance in math, in order to impress their male and female peers. A few, like Summers and Brantley Johnson, even hint that there may be genetic factors at work – that boys may simply have more math ability than girls.

Unfortunately, although there are plenty of diagnoses, there are few proposed cures, ignoring a few well-intentioned but generally unworkable proposals such as all-girl classes throughout high school, an idea that not only has large social and economic implications, but also signals a spineless and unnecessary surrender to the idea that girls cannot compete with boys in the math arena. (Winston Churchill famously said, "Never give in. Never give in. Never, never, never, never, in nothing, great or small, large or petty, never give in."[2] Sometimes it seems that we have adopted this credo wholeheartedly, with only the minor change of eliminating the word "never.")

Failing to find much that was useful in authoritative writings on the subject, I took the next step. I am, among other things, a scientist (a theoretical particle physicist), and thus believe firmly that introspection is no substitute for experimentation and hard evidence. Unfortunately, I was at that time a scientist with a short time fuse and an urgent problem. So, I sat down and thought about the problem, long and hard. And I reached two conclusions, which ultimately became the foundations of the Pittsburgh Plan:

(i) The best way to eliminate the gender gap is to teach girls math, not arithmetic, but real math, very rapidly and very early, before the gender gap exists – thus defining the goals of the Plan; and

1 Although the gender gap is closing by some measures, large disparities remain. For example, boys significantly outnumber girls in higher-level courses such as advanced placement calculus, physics, and computer science, and girls continue to score significantly worse than boys on the math portion of the SAT. In particular, males are between two to three times as likely as females to score between 750 and 800 on the math portion of the SAT. "Total Group Report, 2004 College-Bound Seniors," College Board (2004).

2 From a speech, thought to be largely extemporaneous, given at Harrow School on October 29, 1941.

(ii) If we want to teach real math to young children, we should take advantage of their language-learning ability – thus defining the methods of the Plan.

Let us examine these two foundational ideas in more detail.

The Goal of the Plan:

There is a cultural bias, a stereotype, that girls have inferior math ability. We will not accept this stereotype. Instead, we will beat it. We will mount a full-scale frontal assault on the stereotype by helping our daughters learn math so early and so well that the stereotype will seem laughable when applied to them.

There is little doubt that cultural and environmental pressures on female math students contribute heavily to the math gender gap. Teen Talk Barbie alone is fairly convincing on this point. And one can only imagine the stereotypes that women face in math and science when every "self-respecting physicist and mathematician" – not to mention every science fiction writer - has absorbed Flatland's views on women. Skeptics will point out that Flatland is a social satire and that Abbott's treatment of women is sarcastic and ironic, in much the same vein as Twain's treatment of African-Americans in The Adventures of Huckleberry Finn.[3] Unfortunately, such social and literary subtleties fall on sterile ground when the audience consists of mathematicians, physicists, and science fiction writers!

The self-perpetuating power of this cultural bias is clearly evident in Cynthia Brantley Johnson's quotation set forth at the beginning of this chapter: "For various biological and hormonal reasons, boys seem more suited than girls to deductive reasoning and abstract thought." Without even realizing it, Ms. Johnson has bought into the idea that the ability of females to think in certain key ways is somehow inferior to that of males. And Ms. Johnson is undoubtedly not alone in internalizing this stigma without even realizing it. If a mature and competent professional woman can succumb to the insidious power of this stereotype, imagine the effect that it must have on young girls!

The scope and impact of this negative gender stereotype is revealed by recent comments made by Lawrence H. Summers, former president of Harvard University, an excerpt from which is reproduced at the beginning of this chapter. The gist of Summers' comments is that women are underrepresented in the sciences because (i) they do not want the pressure and intensity of high-level scientific jobs, and (ii) they are genetically less likely to have the necessary brainpower to fill such positions. Summers pooh-poohs the notion that socialization and discrimination are important factors in the gender disparity in the sciences, and concludes instead that "it is impossible to look at this pattern and look at its pervasiveness and not conclude that something of the sort that I am describing has to be of significant importance."[4]

---

3 The Adventures of Huckleberry Finn (Samuel Clemens, 1884).

4 Lawrence H. Summers, Remarks at NBER Conference on Diversifying the Science & Engineering Workforce, Cambridge, MA, January 14, 2005, www.president.harvard.edu/speeches/2005/nber.html.

FIGURE 1.1: Do we really want to send these young girls the message that they are inferior?

Now, let us be honest – although we hope and suspect otherwise, Summers may be right; the jury is still out on the question of whether there are innate differences between the genders in math and science ability. The analytical problem with Summers' remarks is not that he is taking a position that is necessarily wrong; it is that he fails to see that he is making an assumption – in essence, an "act of faith" – in accepting the notion that women are constitutionally inferior to men in the area of high-level scientific thinking ability.

Do you ever wonder why people like Summers say these things? They seem so certain, yet in truth they have no idea whether their views are correct; they have done no careful investigation -- in fact they are opining on a wide-open topic -- and yet they are serene and untroubled by doubt! What mental process allows them to ignore all uncertainty and speak with such conviction? It is as if they think of an idea, consult themselves to test its validity, find unanimity of opinion in favor of the idea, and accept it as truth. This "often wrong but never in doubt" mindset is characteristic of an act of faith – but faith in what? What is the religion to which these uncluttered minds adhere? One almost fears the answer!

The Plan makes the contrary assumption; we assume that women can match men in every respect, but we do so with the full awareness that we are staking out a position in an open area.[5] Summers does not seem to share that awareness, to understand that his conclusion might be wrong. In fact, after making the above statement that it is "impossible" not to agree with him, Summers goes on to introduce a supporting anecdote with the telling phrase, "To buttress conviction and theory with anecdote . . ." The one thing that he does not proffer in support of his conjecture is data. We are given theory, conviction and anecdote — but no data, no facts.

It is tempting to dismiss these statements as the ramblings of a man who has long blurred the distinction between eminence grise and greasy eminence. After all, this is the same Lawrence Sum-

5 There is a third camp that simply refuses to pay any attention whatsoever to the relationship between gender and math, perhaps best exemplified by E. Landau's statement, upon being asked for his testimony to the effect that Emmy Noether was a great woman mathematician: "I can testify that she is a great mathematician, but that she is a woman, I cannot swear." Quoted in J.E. Littlewood, A Mathematician's Miscellany, Methuen and Co Ltd., 1953.

mers that wrote infamously in 1991, as Chief Economist of the World Bank, "I think the economic logic behind dumping a load of toxic waste in the lowest wage country is impeccable ....I've always thought that under-populated countries in Africa are vastly UNDER-polluted . . ."[6]

Unfortunately, such a dismissal would be a mistake. Summers has been an important figure in the corridors of power, and his brief moment of candor reveals the depth of the negative gender stereotype at the highest levels of power and influence. Whatever may be said publicly, in their private moments, as they consider and decide matters such as tenure and promotion, the old white guys in the suits are influenced by this prejudice.

Although we cannot dismiss the people who embrace these types of stereotypes, we can certainly spare a moment to disrespect them! In truth, these bigots are almost universally people who (i) lack personal qualifications in the area in question, and (ii) are members of the class said by the stereotype to be superior. For example, Summers, who believes that men are better scientists than women, is an economist, not a scientist. (Economists are sometimes referred to as "social scientists," but have about the same relation to true scientists as a "clothes horse" has to a true horse).

I have personally witnessed the pervasive and often subtle gender bias in the world of mathematics on many occasions. I first noticed it as a senior in high school, when I was vying with a female classmate for top honors in our calculus class. We were neck-and-neck for three or four tests, but she did very well on the fifth test and pulled into the lead. When the teacher returned the tests, he announced our scores and then said, as he was handing me my test, "Are you going to let a girl beat you?" He said this jokingly, not with any bad intent, and the whole class laughed.

Well, almost the whole class. I did not laugh, because I was annoyed. I remember thinking, "What does that have to do with anything? She is really smart." And one other person did not laugh – the girl. I looked at her and was amazed to see that she looked embarrassed and sad. Perhaps it was just a coincidence, but from then on her performance in that class was completely average. She disappeared into the middle of the class. For thirty years I have wished that I had said something to her . . . .

Years later, shortly after my first daughter was born, I was sitting in my physics graduate student office with two other graduate students, Sarah and Fred, when one of our fellow graduate students, Dwayne, walked in, and the following conversation took place:[7]

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6 Lawrence Summers, Internal Memorandum, December 12, 1991, see "Let Them Eat Pollution," Economist (London), February 8, 1992, p. 66. Apparently this memorandum was actually written by an assistant and authorized by Summers to be distributed under Summers' signature. Summers has claimed that the memorandum was intended to be ironic; as in the case of Summers' recent statements about women and science, the contents of the memorandum are incendiary but not necessarily false.

7 Names have been changes to protect the innocent and the guilty.

Dwayne: "Fred, can you help me with the quantum mechanics homework?"
Fred: "I'm pretty busy right now."
Sarah: "Dwayne, I did that problem. I could help you."
Dwayne: "Rich, did you get that problem?"
Me: "Sorry, I haven't done the homework yet." [8]
Sarah: "I did that problem."
Dwayne: "Well, Fred, could I just ask you a quick question about it?"
Fred: "No, I really am too busy. Talk to Sarah; she solved it."
Dwayne: "I'll just come back later." [Departs]

Although Fred and I commiserated with Sarah about Dwayne's behavior, she remained very upset. It is easy to think from the outside that people should be tough and ignore such things, but that kind of thinking is not analysis, it is a failure of imagination. For people who live constantly with this sort of stereotyping, which is really just another word for discrimination, the cumulative effect can be enormous.

So, let us set a ground rule right from the outset. For all we know, Cynthia Brantley Johnson may be correct; females may have less ability in deductive reasoning and abstract thought than males. But the jury is still out on this question, and unless and until the evidence compels us to conclude otherwise, we will operate under the assumption - and expectation! - that females are in every sense the equal of males in these areas. The Pittsburgh Plan does not accept the stereotype. And, perhaps as a result, the Plan's female participants have shown no evidence at all of any sort of math gender gap.[9]

> **". . . we will operate under the assumption - and expectation! - that females are in every sense the equal of males in these areas."**

A brief digression: One of the guiding principles of the Pittsburgh Plan is that we tend to get what we expect. If we expect that children cannot learn long division until fourth grade, they will not learn long division until fourth grade. Why? Because we will not try to teach it to them until fourth grade. Children in the Pittsburgh Plan usually learn long division in first or second grade. Why? Because we are not allowing our expectations to set artificial limits; we allow them to learn long division when they are ready, which usually turns out to be age six or seven.

8 This is probably the phrase that I uttered most often while in physics graduate school.

9 Girls in other countries do substantially better than American girls on math performance, suggesting that at least a portion of the American math gender chasm is cultural rather than biological. E.g., in the 2003 TIMSS study of eighth-grade mathematics performance, gender differences were negligible in many countries, girls significantly outperformed boys in nine countries, and boys significantly outperformed girls in nine countries. The results of the TIMSS 2003 assessment of fourth-grade mathematics performance were similar. These results are wildly at odds with Lawrence Summers' "nature over nurture" views!

The same principle applies to the idea of a female stereotype. If we expect girls to perform poorly, they will internalize that expectation and fulfill it, even if the stereotype is not otherwise true. If we expect girls to do well, it turns out that they do well.[10]

Are we just talking through our collective hats? Do we have any evidence at all for this broad, sweeping statement? Sure we do, reams of it. Just as one example, consider the proof set forth in Figure 1.2, done by a fifteen-year-old graduate of the Plan. The proof, that the rational numbers Q are not G_δ, was done for extra credit in her Real Analysis course at Carnegie Mellon University – a course that is at least three courses beyond the usual AP Calculus courses taken by the most advanced fifteen-year-old high school students.

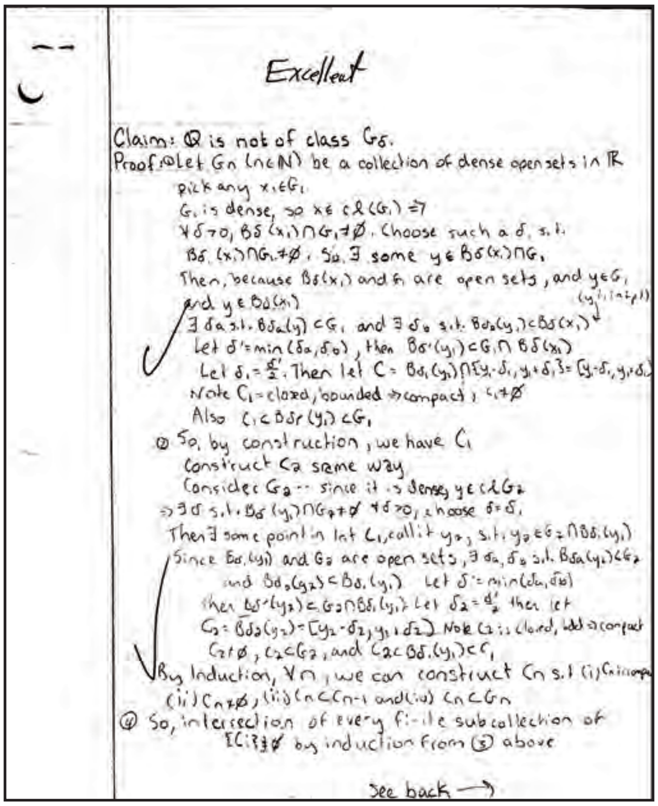
Given that there is a pervasive gender bias, an important part of our task is to figure out ways to provide girls with self-confidence, strength of character, and other forms of interpersonal armor. In fact, the early competency that comes from participation in the Pittsburgh Plan is a powerful force in this direction.

But, at its core, such skin-thickening is a defensive approach. Why not set aside our preconceptions and meet this problem head on? Why not take the offensive? What would happen if our daughters learned so much math in their early years that they were experts by the time the cultural pressures settled in? Is it possible that they could learn so well, and so rapidly, that instead of hunker-ing down and finding ways to handle the pressure, they could just blast through the pressure?

In 1991 I did not know the answer to these questions. But I resolved to try to find out. I would structure a plan for very early, very rapid learning of mathematics. And, in case this was too ambitious a goal, I would pursue this plan in a low-key, pressure-free way, so that my young daughters would not suffer if it turned out that we were striving for the impossible, or even the highly unlikely. In short, the goals of the Plan – rapid early learning, no pressure – sprang from our unwillingness to accept the pernicious gender stereotype described above.

Although the goals of the Plan were originally defined in the context of the negative gender stereotype and the gender gap in math, we quickly began to consider whether these goals made sense for all children, regardless of gender. For example, it appears that there is a significant decline in the percentage of all students, both boys and girls, who vigorously pursue mathematics between fourth grade and twelfth grade -- could the same technique of early and rapid learning could help reverse this negative trend? We will explore some of these ideas further in the following chapters; for the moment, it is sufficient to note that the Pittsburgh Plan adopts the goal of rapid early learning of math equally and neutrally for both boys and girls.

10 Although "we get what we expect" is easy to say and understand, it is only an approximation of the actual philosophy of the Pittsburgh Plan. In fact, it is our view that both negative and positive expectations can create problems, negative expectations because they limit the opportunities we give our children (as described in the text), and positive expectations because they can create pressure to perform. The true Pittsburgh Plan approach is to avoid expectations altogether, and simply let the child go where her abilities and interests take her.



Excellent

Claim: $\mathbb{Q}$ is not of class G_δ.

Proof: ① Let G_n ($n \in \mathbb{N}$) be a collection of dense open sets in $\mathbb{R}$

pick any $x_1 \in G_1$

G_1 is dense, so $x \in cl(G_1) \Rightarrow$

$\forall \delta > 0$, $B_\delta(x_1) \cap G_1 \neq \emptyset$. Choose such a δ, s.t.

$B_\delta(x_1) \cap G_1 \neq \emptyset$. So $\exists$ some $y \in B_\delta(x_1) \cap G_1$

Then, because $B_\delta(x_1)$ and G_1 are open sets, and $y \in G_1$

and $y \in B_\delta(x_1)$

$\exists \delta_a$ s.t. $B_{\delta_a}(y) \subset G_1$ and $\exists \delta_b$ s.t. $B_{\delta_b}(y_1) \subset B_\delta(x_1)$

Let $\delta' = \min(\delta_a, \delta_b)$, then $B_{\delta'}(y_1) \subset G_1 \cap B_\delta(x_1)$

Let $\delta_1 = \frac{\delta'}{2}$. Then let $C_1 = B_{\delta_1}(y_1) \cap [y_1 - \delta_1, y_1 + \delta_1] = [y_1 - \delta_1, y_1 + \delta_1]$

Note C_1 = closed, bounded $\Rightarrow$ compact; $C_1 \neq \emptyset$

Also $C_1 \subset B_{\delta'}(y_1) \subset G_1$

② So, by construction, we have C_1

Construct C_2 same way

Consider G_2 -- since it is dense, $y \in cl G_2$

$\Rightarrow \exists \delta$ s.t. $B_\delta(y_1) \cap G_2 \neq \emptyset$ $\forall \delta > 0$, choose $\delta = \delta_1$

Then $\exists$ some point in Int C_1, call it y_2, s.t. $y_2 \in G_2 \cap B_{\delta_1}(y_1)$

Since $B_{\delta_1}(y_1)$ and G_2 are open sets, $\exists \delta_a, \delta_b$ s.t. $B_{\delta_a}(y_2) \subset G_2$

and $B_{\delta_b}(y_2) \subset B_{\delta_1}(y_1)$. Let $\delta' = \min(\delta_a, \delta_b)$

then $B_{\delta'}(y_2) \subset G_2 \cap B_{\delta_1}(y_1)$. Let $\delta_2 = \frac{\delta'}{2}$ then let

$C_2 = \overline{B_{\delta_2}(y_2)} = [y_2 - \delta_2, y_2 + \delta_2]$. Note C_2 is closed, bdd $\Rightarrow$ compact

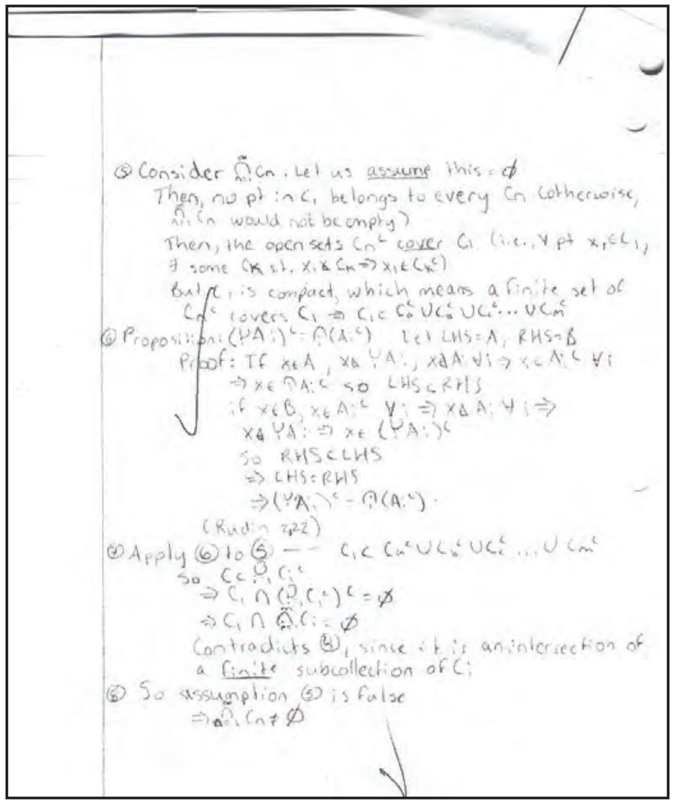
$C_2 \neq \emptyset$, $C_2 \subset G_2$, and $C_2 \subset B_{\delta_1}(y_1) \subset C_1$

By Induction, $\forall n$, we can construct C_n s.t. (i) C_n is compact

(ii) $C_n \neq \emptyset$, (iii) $C_n \subset C_{n-1}$ and (iv) $C_n \subset G_n$

④ So, intersection of every finite subcollection of $\{C_i\} \neq \emptyset$ by induction from ③ above

See back ⟶



⑤ Consider $\bigcap_1^\infty C_n$. Let us assume this $= \emptyset$

Then, no pt in C_1 belongs to every C_n (otherwise, $\bigcap C_n$ would not be empty)

Then, the open sets C_n^c cover C_1 (i.e., $\forall$ pt $x_1 \in C_1$, $\exists$ some C_k s.t. $x_1 \notin C_k \Rightarrow x_1 \in C_k^c$)

But C_1 is compact, which means a finite set of C_n^c covers $C_1 \Rightarrow C_1 \subset C_{\alpha_1}^c \cup C_{\alpha_2}^c \cup \ldots \cup C_{\alpha_m}^c$

⑥ Proposition: $(\bigcup A_i)^c = \bigcap (A_i^c)$ Let LHS = A, RHS = B

Proof: If $x \in A$, $x \notin \bigcup A_i$, $x \notin A_i \ \forall i \Rightarrow x \in A_i^c \ \forall i$

$\Rightarrow x \in \bigcap A_i^c$ so LHS $\subset$ RHS

if $x \in B$, $x \in A_i^c \ \forall i \Rightarrow x \notin A_i \ \forall i \Rightarrow$

$x \notin \bigcup A_i \Rightarrow x \in (\bigcup A_i)^c$

So RHS $\subset$ LHS

$\Rightarrow$ LHS = RHS

$\Rightarrow (\bigcup A_i)^c = \bigcap (A_i^c)$.

(Rudin 2.22)

⑦ Apply ⑥ to ⑤ -- $C_1 \subset C_{\alpha_1}^c \cup C_{\alpha_2}^c \cup C_{\alpha_3}^c \ldots \cup C_{\alpha_m}^c$

So $C_1 \subset (\bigcap C_i)^c$

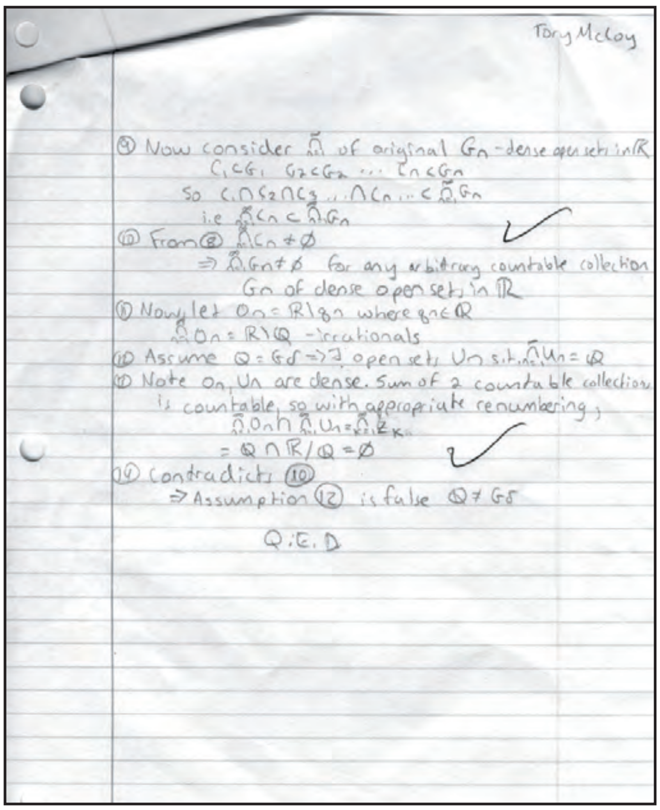
$\Rightarrow C_1 \cap (\bigcap C_i)^c = \emptyset$

$\Rightarrow C_1 \cap \bigcap C_i = \emptyset$

Contradicts ④, since it is an intersection of a finite subcollection of C_i

⑧ So assumption ⑤ is false

$\Rightarrow \bigcap_1^\infty C_n \neq \emptyset$



Tory McCoy

⑨ Now consider $\bigcap_1^\infty$ of original G_n - dense open sets in $\mathbb{R}$

$C_1 \subset G_1$, $C_2 \subset G_2$ ... $C_n \subset G_n$

So $C_1 \cap C_2 \cap C_3 \ldots \cap C_n \ldots \subset \bigcap_1^\infty G_n$

i.e. $\bigcap_1^\infty C_n \subset \bigcap_1^\infty G_n$

⑩ From ⑧ $\bigcap_1^\infty C_n \neq \emptyset$

$\Rightarrow \bigcap_1^\infty G_n \neq \emptyset$ for any arbitrary countable collection G_n of dense open sets in $\mathbb{R}$

⑪ Now let $O_n = \mathbb{R} \setminus q_n$ where $q_n \in \mathbb{Q}$

$\bigcap O_n = \mathbb{R} \setminus \mathbb{Q}$ - irrationals

⑫ Assume $\mathbb{Q} = G_\delta \Rightarrow \exists$ open sets U_n s.t. $\bigcap_1^\infty U_n = \mathbb{Q}$

⑬ Note O_n, U_n are dense. Sum of 2 countable collections is countable, so with appropriate renumbering,

$\bigcap O_n \cap \bigcap U_n = \bigcap R_k$

$= \mathbb{Q} \cap \mathbb{R}/\mathbb{Q} = \emptyset$

⑭ Contradicts ⑩

$\Rightarrow$ Assumption ⑫ is false $\mathbb{Q} \neq G_\delta$

Q.E.D

FIGURE 1.2: The former president of Harvard University claims that girls are inferior at math. The fifteen year-old author of this proof, might disagree. (The proof was done for extra credit in her Real Analysis course at Carnegie Mellon University.)

The Method of the Plan:

It seems likely that the innate language learning mechanism possessed by young children can be very helpful in learning math. This suggests that we structure our math-learning approach to mimic the conditions in which language is acquired: immersion in substantial content of mixed levels, coupled with significant parent-child interaction, with a heavy reliance on induction by the child.

In this section, we lay out a few theoretical speculations that have guided the Plan or have been prompted by its successes. Please note that the Plan is heuristic in nature; it stakes its claim based solely on its results. We hope that these speculations are interesting and help provide insights into the "tone" of the Plan, but emphasize that we are not attempting to engage with educational orthodoxy at a theoretical level. As mentioned earlier, our sole response to the skepticism of teachers, psychologists and other experts is a proffer of our results.

Although it seems clear that cultural gender bias plays a large role in the math gender gap, it is important that we not dismiss other possible explanations – e.g., an innate difference between males and females in math ability or, alternatively, in math-learning strategies - simply because they are uncomfortable or provoke visceral negative reactions. In a matter as crucial as the education of our children, the importance of a correct understanding outweighs the benefits of coddling cherished viewpoints.

Thus, although we refuse to accept the stereotyping of females unless and until there is compelling evidence to support the stereotype - and to date there is no such evidence - we do not close our eyes to evidence of possible biological factors contributing to the math gender gap. Instead, we actively seek such evidence, because it will help us structure a better approach to bridging the gap.

One fact immediately leaps to mind: the nature of the math curriculum changes dramatically in middle school. In elementary school, the curriculum consists almost entirely of arithmetic in different manifestations – elementary number operations, fractions, decimals, etc. In middle school, there is suddenly more emphasis upon abstract thinking, especially as children begin to study algebra. These new types of task require a very different set of abilities than the elementary school tasks. In fact, it is fair to say that American schools do not begin teaching mathematics, as opposed to routine arithmetic, until middle school. Could it be a coincidence that girls' problems with math begin at the same time that the nature of the math curriculum changes fundamentally?

It is well known that humans have an innate language-learning ability that seems to disappear in the teen years.[11] Children learn languages with astonishing ease; adults cannot hope to match that

11 This idea, which is now widely accepted, shook the world when it was first introduced by Noam Chomsky as an alternative to cognitive (Piaget) and behavioral (B. F. Skinner) explanations of language.

level of language proficiency. This innate ability seems to have a large component that helps children internalize and master the underlying syntax of a language. But, mathematics is in a very real sense a language (or, perhaps more accurately, a set of languages); certainly, it has its own underlying set of syntactic rules, and in fact different branches of mathematics – geometry, basic algebra, topology, group theory, real analysis, etc. – can be thought of as different languages, layering different surface syntactic rules over common underlying semantic content. Thus, the innate language mechanism might well be very helpful in the mastery of mathematics. Could it be that the problems high school girls have with math are in some sense related to the disappearance of this innate language-learning ability? Does the slower process of physical maturation in boys result in a slower or delayed loss of the innate language-learning ability, making that key ability more available to boys than girls during the first few years of real mathematics instruction?

Or, alternatively, is it possible that girls and boys learn math differently, even at very young ages? Could it be that boys learn math spatially, while girls learn math as a language? [12] If so, girls might well be disproportionately affected by the disappearance of the innate language learning mechanism during adolescence. There seems to be no evidence that spatial-type intellectual abilities diminish in the same way that the innate language ability does, although every physicist and mathematician is well aware that innovative work in their fields must generally be done in the early part of one's career. Still, this gives spatial learners a decided advantage over language-based learners.[13]

What do these ideas mean for our purposes, for the development of a plan to bridge the math gender gap? Suppose we want to tap into the innate language learning mechanism for math instruction. How should we go about it? Should we not mimic the environment in which a child learns language – immersion, coupled with a certain level of instruction? Why not let children learn the rules of math in a problem-based, inductive way? And, why not have them learn it through positive interactions with their parents, just as they learn English in their early years?

This means that we should not be too worried if we give children problems that they cannot solve, problems that involve concepts and techniques they have not yet learned. In fact, we want them to learn inductively, with guidance. Anyone who has taught math or physics knows that (i) no one really understands a concept until he or she has used it to solve problems, and (ii) familiarity and mental comfort with a topic greatly improves a student's learning curve. For example, the physical principles describing electromagnetism are at the elementary level almost identical to those describing gravity, and at the more advanced level are simpler. Yet, students have a much harder time with electromagnetism than gravity, possibly because of their everyday, intuitive experience with gravity.

12 The widest boy-girl disparity on the SAT occurs in geometry questions.

13 Another observed male-female difference occurs at the extreme ends of the math performance scale: males are far more likely than females to score at the very top or very bottom of the scale. We have not been able to extract any useful principles for the Pittsburgh Plan from this higher variability of males; perhaps as you implement the Plan you will see something that we did not.

This suggests that it might be beneficial to introduce difficult concepts at an early age, even if the student will not fully understand them at that time.

In summary, the Pittsburgh Plan rejects the negative female stereotype, introduces math concepts early and fast, and structures homework sessions to mimic the environment in which natural language learning takes place. And if Mattel were to introduce a "Pittsburgh Plan Barbie," it would say:

"Math is fun. And I still love shopping."

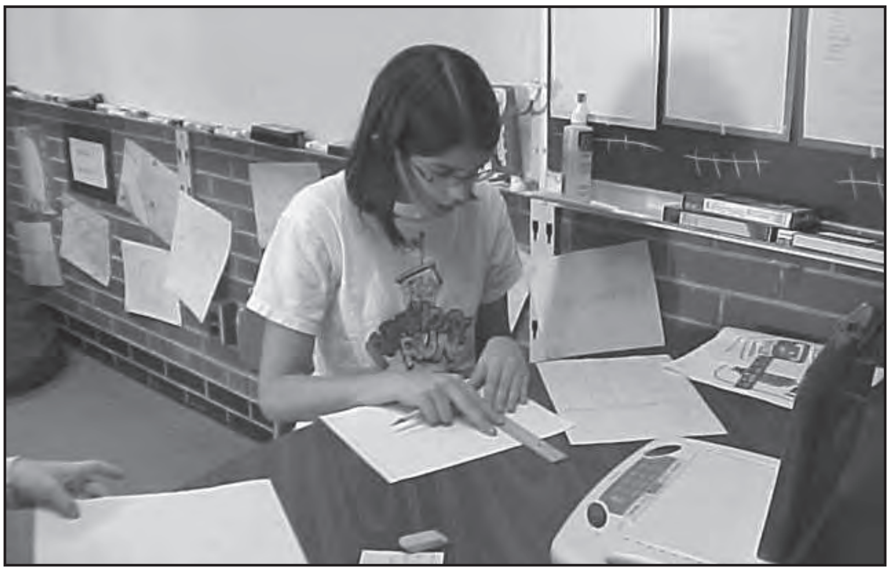


FIGURE 1.3: A Plan participant hard at work in her fifth-grade classroom. She graduated from the Plan in the summer after fifth grade, took AP Cal-culus at the local high school in sixth grade, then began taking advanced math courses at the University of Pittsburgh and Carnegie Mellon University in seventh grade.

Chapter 2: *Crossing the Class Chasm*

I saw the best minds of my generation destroyed by madness,
starving hysterical naked . . . .
who poverty and tatters and hollow-eyed and high
sat up smoking in the supernatural darkness of cold-water flats. . . .
who cowered in unshaven rooms in underwear,
burning their money in wastebaskets
and listening to the Terror through the wall . . . .

Allen Ginsberg, "Howl," 1956

School's out for summer
school's out forever . . .
Out for summer out till fall
we might not come back at all.

Alice Cooper, "School's Out," **1972**

There are two class gaps in American math education: the gap between the poor and everyone else; and the gap between the rich and everyone else.

The first gap, the gap between the poor and everyone else, is the more serious. In fact, it is not so much a gap as a chasm. From their very first days in school, poor children vastly under-perform on all measures of math achievement (and reading achievement, and pretty much every other type of academic achievement).

How bad is the problem? Approximately fifty percent of poor students do not meet minimum standards of math performance. By contrast, only twenty percent of other children fail to meet these minimum standards.[1] By the end of fifth grade, poor children fall more than two years behind other students in verbal achievement and one and one-half years behind in math. Twelfth-grade students who are eligible for the national school lunch program, i.e., who are poor, perform at about the same math level as 8th-grade students who are not in the school lunch program.

One could go on and on reciting statistics like these, but the point should be pretty clear. In our land of opportunity, a vast number of poor children emerge from the school system with almost no chance of reaching the middle class.

What can the schools do to attack this terrible problem? The sad truth is, not much. There

1 Actually, twenty percent is horrible too, just not as horrible as fifty percent.

is growing evidence that this particular problem has its roots in the home rather than the school. For example, the statistics concerning underperformance by poor children are similar across all grades, meaning that the class chasm is present in full force by the time poor children begin school. Also, to the extent the class chasm widens during the school years, that widening apparently can be attributed almost entirely to the cumulative effect of summer learning differences. Although it is normal for students take a step backward during the summer months, losing at least some academic ground, poor students suffer far greater summer losses than do other students. [2]

Let us ask a different question. What can we do to help poor children cross the chasm? [3] Several Johns Hopkins University researchers suggest an answer: "If we could get poorer children up to speed before they start school, perhaps schools could do even more to close the achievement gap." [4] In other words, we should teach our children math and reading early, before they begin school. And, if summer learning loss is a problem, perhaps we should teach our children during the summer, too. In short, we should follow the Pittsburgh Plan!

"If you are reading this book and you are poor, you have come to exactly the right place."

Of course, it would be disingenuous to suggest that the Pittsburgh Plan is the answer to the problem of the class chasm in reading and math. To begin with, the Plan assumes heavy parental involvement, which may not be realistic for some poor children. More importantly, many seriously poor children face staggering problems at home – e.g., lack of food and shelter – that would not even be touched by the Plan. Nonetheless, in many cases the Plan can provide poor children with an early boost and a way to maintain their academic gains over summer vacations. But, these are generalities; let us cut to the chase. If you are reading this book and you are poor, you have come to exactly the right place. The Pittsburgh Plan can help your child.

What about the second class gap in American math education, the gap between the rich and everyone else? Wealth provides a well-paved road to academic success for two basic reasons: (i) rich children generally receive a better education than their middle class peers, and thus develop greater competency; and (ii) among well-qualified students, rich children are given preferential access to premier schools and academic opportunities.

If you are wealthy, this state of affairs is probably just fine with you. But if you are not wealthy, you may resent these advantages flowing to wealthy children. Certainly, we all know as adults that

2 Johns Hopkins University, Center For Summer Learning Website, Research Tab (2002) , citing research by faculty members Karl Alexander and Doris Entwisle; www.jhu.edu/teachbaltimore/resourcesresearch/sumloss.html

3 The phrase "crossing the chasm" was coined by Geoffrey Moore in his insightful book, Crossing the Chasm: Marketing and Selling Technology Products to Mainstream Customers (Harper Collins, 1991).

4 Doris R. Entwisle, Karl L Alexander, and Linda Steffel Olson, "Keep the Faucet Flowing: Summer Learning and Home Environment,"The American Educator (Fall 2001).

life is not fair, that the playing field is not level. But we treasure the idea, the American Dream, that our children will have a fair chance to build a good life. In this land of opportunity, we trust that our children will be judged on their talent and their hard work, and will have the same chance as everyone else to get a good education that will open doors to good careers.

Unfortunately, it just does not work this way, as I discovered first-hand when my children were born. I had abandoned my career as a Los Angeles attorney to pursue a doctorate in theoretical particle physics at the University of Virginia, but I kept in close touch with my many Los Angeles friends, most of whom had family incomes ranging from twenty to fifty times the amount that my wife and I were then earning. I had never regretted my decision to abandon the high-income lifestyle until I realized the extent to which my friends' children were benefiting from their wealth – i.e., the extent to which I had placed my children at a competitive disadvantage for top-flight schools and jobs. To be sure, as solid members of the middle class, my children would have every opportunity to get decent jobs and have decent careers, but I did not want to settle for that. I wanted to give them a fair opportunity to do even better than that, to do great things, to achieve their dreams.

But, I soon realized that my children were facing an uphill climb. While they would be going to standard, respectable public schools, my friends' children were attending elite prep schools such as the famous Harvard Westlake school in Los Angeles, California. Does a school like Harvard Westlake provide a real advantage? Well, you decide for yourself.

At the time of this writing, Harvard Westlake has 1,553 students and 211 faculty members, roughly one faculty member for every eight students. More than half of the faculty members hold masters degrees and twenty-five faculty members (about 12%) hold doctoral degrees. The school has eight academic halls, two lecture halls, two libraries, three gymnasiums, two pools, two football/soccer fields, a track, two auditoriums, two orchestra rooms, three instrumental rehearsal studios, two electronic music studios and four additional fully-equipped MIDI stations, music practice rooms, two dance studios, two arts centers, two ceramic studios, two photography labs, and computer labs with terminals throughout each campus. Harvard Westlake students average 688 on the SAT Verbal and 704 on the SAT Math, compared with national averages of 500 on each test.[5] Believe me, you will look long and hard before you find a public school that can come close to matching these statistics! What does this kind of quality cost? In the case of Harvard Westlake, about $22,000 per year.

We need to be clear that there is nothing at all wrong with this. Harvard Westlake is providing a wonderful education for its students, and that is a good thing. The world would be a better place if every school were like Harvard Westlake! But you should be aware that your child is not playing on a level field, at least not if he or she wants to strive for an extraordinary academic career. Perhaps you are as surprised as I was at the magnitude of this gap. And perhaps you want to fight back on behalf

5 All information is taken from Harvard Westlake's Web site, www.harvardwestlake.com (updated 8/26/03).

of your child. (We will get to that a little later.)

There is a darker side to this gap, an aspect that is not necessarily fair or appropriate. Many of the elite prep schools – and I want to make it clear that I am not singling out Harvard Westlake at this point; in fact, I do not know whether Harvard Westlake does this kind of thing or not – apparently use a shadowy network of connections and influence to ensure that their students have an advantage in the college admission process. This is not the type of advantage that might come from a better education – that is just part of the deal, and if a rich child can become smarter and more knowledgeable due to his economic advantages, well, then, maybe he does deserve to go to the better schools. This is something more sinister, namely, the manipulation of the college admission process so that wealthy students are admitted ahead of equally qualified but less privileged students. To put it bluntly, the elite American prep school system allows wealthy parents to buy your children's college slots for their own children. So much for the American Dream in education!

"... the elite American prep school system allows wealthy parents to buy your children's college slots for their own children."

How is this done?

First, as noted above, these elite prep schools have excellent facilities, faculty and student-teacher ratios. As a result, they do a better job of educating their students. There is nothing wrong with this – in fact, it is commendable! – but the corollary is that your child is at a competitive disadvantage coming out of high school.

Second, these private schools do an excellent job of preparing their students for the SATs and the college admission process, sometimes even offering sophisticated classes or programs on topics such as SAT preparation and writing a college application essay. As a result, the SAT and the application essay are not truly impartial tests of ability, but instead reflect differences in task-specific preparation. And, one cannot help but wonder to what extent college admission essays by prep school students are the result of a collaborative process between the wealthy students and their elite instructors.

Third, the high tuition at these private schools (and the corresponding low student- teacher ratio) in effect buys several strong teacher recommendations, written by teachers who are expert at the recommendation process and in fact may have received institutional guidance from the school on how best to prepare recommendations.

Fourth, as noted above, many of these schools cultivate contacts with admissions offices at good colleges so that their students have a better chance of receiving favorable treatment. In fact, these private schools are often quick to trumpet their close connections with Ivy league admissions offices, and many routinely schedule eleventh grade "field trips" to Ivy League colleges.

The point here is that, unbeknownst to you (until now), even if you are solidly ensconced in the middle class, your children are under attack. Wealthy parents are spending vast amounts of money and enlisting top-flight teaching talent in an effort to help their children take your children's spots in good colleges. They are taking dead aim at your children.[6] And they are not fighting fair, just using their money to build a better student. Instead, they are also using their money to gain procedural advantages in the college admissions process. Now, it is important to note that this is entirely legal, and also entirely understandable. What parent would not use his wealth to help his child get a good start in life? The question is not whether wealthy parents are within their rights in giving their children these competitive advantages; the question is whether you will take this lying down, or instead find a way to fight back on behalf of your child.

There is another aspect to the "rich versus everyone else" class gap, a sort of resonance effect where initial success, which can often be purchased, serves to get a child identified as being gifted, which then qualifies her for a host of special programs and opportunities. Often these programs are not going to make a big difference in the child's education, but they cannot hurt. And, the children who qualify for such programs are recognized by their peers as being among the intellectual elite, and thus develop a self-image that helps them perform at their peak potential.

"The question is not whether wealthy parents are within their rights in giving their children these competitive advantages; the question is whether you will take this lying down . . ."

Also, there are many extracurricular programs that focus on math education for "gifted children." For example, the Center for Talented Youth at Johns Hopkins University provides a variety of excellent summer residential and day programs, as well as full-year distance learning programs, for students who qualify as gifted under standardized testing. Such programs often require computers, Internet connections and/or the payment of high tuition --in effect, wealth-based selection criteria.[7] The emphasis on gifted children also has a hidden class bias, because there is no doubt that "giftedness," as measured at the elementary school level, can to some extent be purchased; rich children, who get small classes, good teachers, tutors, summer programs, private testing by psychologists hired by the child's parents, and the like, have a much better chance of qualifying as gifted than do their less wealthy contemporaries who cannot afford such "head start" mechanisms.

6 "At prestigious universities around the country, from flagship state colleges to the Ivy League, more and more students from upper-income families are edging out those from the middle class . . . . Experts say the change in the student population is a result of both steep tuition increases and the phenomenal efforts many wealthy parents put into preparing their children to apply to the best schools." David Leonhardt, "As Wealthy Fill Top Colleges, Concerns Grow Over Fairness,"The New York Times, April 22, 2004.

7 The Johns Hopkins University programs make every effort to counteract this unintended effect – which in some sense is an unavoidable by-product of offering distance learning programs -- through a vigorous need-based scholarship program.

How do you help your kids compete with children who go to elite, well-funded schools with rich enrichment opportunities, schools like Harvard Westlake? How can you counter the effects of up to twelve years of superior schooling? It seems to be a hopeless endeavor!

I must confess that after a while I grew tired of hearing my friends' well-meaning stories about the wonderful educations that their children were receiving. Not only was I upset at the idea that I had forfeited important opportunities for my children, I was also put off by the unconscious assumption that children who were succeeding in the nurturing embrace of an elite prep school were superior to children (like my own) who did not have such advantages. It seemed to me to be a classic case of confusing being born on third base with hitting a triple.

". . . I realized that you and I have everything we need to give our children the advantage in this struggle."

Ultimately, I decided to fight back. So I thought for a while, and then I realized something very important. I realized that you and I have everything we need to give our children the advantage in this struggle. The very best education does not require a computer, a tutor, a graphics calculator, or an enrichment course at school. It requires three things: a pencil; lots of paper; and a parent who is willing to turn off the television and spent a few minutes reading and doing math with her child. You can do all of that! It does not take money; it does not even take math knowledge; it just takes a strong resolve.

And so I developed the Pittsburgh Plan, first for myself and my children, but also for you and your child. Under the Plan, you will work with your child at an early age, where your math knowledge will be far in excess of what is needed. By the time your child is old enough to test the limits of your math expertise, your main work will be done. Her course will be charted; she will be on the path to success.

Again, this is not a fancy program. We do not have CDs with great graphics; we do not have interactive computer programs that monitor a child's performance to determine the appropriate lesson plan; we do not even have teachers who have Ph.D.'s in child psychology and education. We have pencils, paper, and parents who are willing to spend time on their children. We have a math program designed by a parent who really understands math and children. And we do not discriminate; if you want to play our game, we welcome you. We do not care if you are rich or poor, and we do not care whether your child meets some psychologist's definition of "gifted." We believe every child can do math, and we are confident that we can help every child improve his or her math performance.

The funny thing is – our program is superior to the fancy ones. No calculator can help a child learn math as well as a pencil can, and no mentor, or computer program, or sophisticated enrichment class, can equal time with a parent. In fact, it turns out that this gap actually works in the opposite direction; all the Harvard Westlakes in the world cannot match the time and attention of a loving parent.

These ideas have a higher-education corollary: the current emphasis upon getting one's child into an "elite" college – which can dominate parents' thoughts as early as kindergarten -- is more the result of mass hysteria than rational analysis. Why do we say that? First, the quality of an individual's college education is almost entirely determined by his or her effort, determination and ability. Will Hunting is exaggerating, but is not entirely off the mark, when he says to an arrogant Harvard student: "Fifty years from now you're going to do some thinking on your own and realize . . . that you wasted $150,000 on an education that you could have gotten for $1.50 in late charges at the public library." [8] Second, although the reputation associated with an elite-school degree can provide a graduate with an initial advantage in entering the workforce or graduate school, the magnitude of this advantage is relatively small when compared with (i) the human cost of single-minded pursuit, for many years, of admission to an elite college and (ii) the actual economic cost of an elite college education. The advantage may not even exist at all vis-à-vis students who excel at a non-elite school.

". . . all the Harvard Westlakes in the world cannot match the time and attention of a loving parent."

In short, Grasshopper, do not pursue a reputation;
pursue competence, and let the reputation follow.

8 Good Will Hunting (Miramax Pictures, 1997).

CHAPTER 3:
Spanning the International Gulf

"For we must Consider that we shall be as a City upon a Hill, the eyes of all people are upon us…."

--- **John Winthrop**, 1630

"Let the word go forth from this time and place, to friend and foe alike, that the torch has been passed to a new generation of Americans—born in this century, tempered by war, disciplined by a hard and bitter peace, proud of our ancient heritage . . . . Let every nation know, whether it wishes us well or ill, that we shall pay any price, bear any burden, meet any hardship, support any friend, oppose any foe . . ."

--- **John F. Kennedy,** Inaugural Address, January 20, 1961

"By the year 2000, United States students will be the first in the world in mathematics and science achievement."

--- **National goal for math education in the U.S.,**
adopted by the President and Governors, 1990

"Compared with students in 37 other participating nations, US 8th-grade students are above the international average in mathematics and science performance . . . . "

--- **Triumphant U. S. Department of Education Press Release**, December 5, 2000

So it has come to this . . . . the greatest nation in the history of the world, the City upon a Hill, now celebrates when its students are "above the international average" in math and science.

How far above average, you ask? First place? Second place? That is a fair question, but you will not find the answer to it in the press release cited above. Instead, you have to dig into the actual report, known as the Third International Mathematics and Science Study–Repeat (TIMSS–R).
If you take the trouble to read the report, you will find the cause for this celebration: American students placed nineteenth out of thirty-eight nations in math, and eighteenth in science.

Only a politician calls this better than average; for everyone else this is an average performance. And only a politician, namely, former U.S. Secretary of Education Richard W. Riley, would react to this news by saying, "Our students are successfully learning more math and science every year they're in school." This is reminiscent of the old cereal commercials that used to advertise "99 %

more vitamins!" 99% more than what? Grass? Dirt? Read literally, Riley is making the brave claim that our students know more math at the end of a school year than they did at the beginning of the year. Certainly we all hope this is true, but it seems to be a very modest aspiration.

Riley goes on in the press release to say, "but we can do even better." Let us hope so! After all, we finished nineteenth out of thirty-eight. And in order to do even this well, we stacked the deck: we changed the comparison countries dramatically from the original 1995 study, where the United States did even worse. For example, in the 1999 study we eliminated countries such as Denmark, France, Germany, Ireland, Norway, Belgium-French, Sweden and Switzerland – all of whom had outperformed the United States in the earlier 1995 study -- and replaced them with countries such as Jordan, Indonesia, Chile, the Philippines, Morocco, Turkey, Tunisia, and Moldova, all of whom performed worse than the United States in the new study. [1]

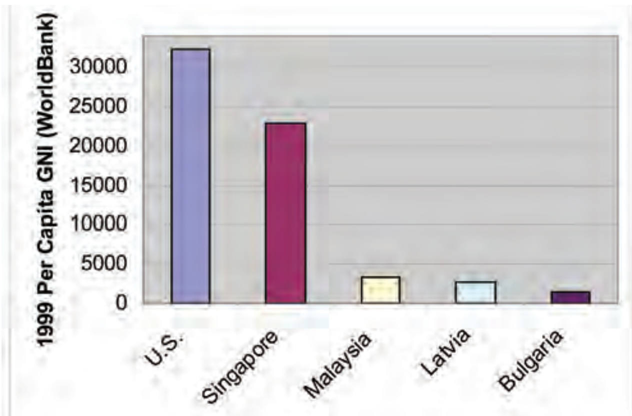
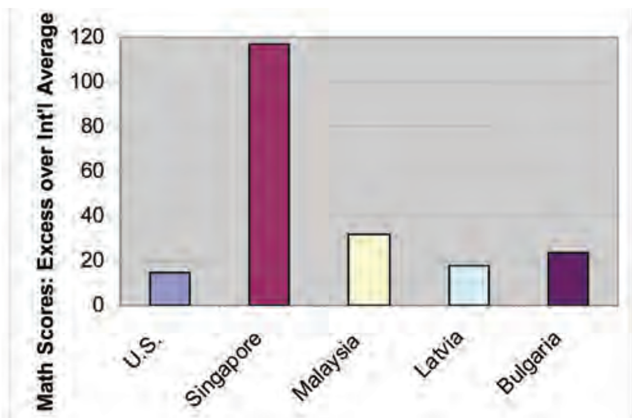


FIGURE 3.1: When compared on the basis of 1999 Gross National income per person, the United Statees looms large!



But when compared on the basis of excess of 1999 TIMSS math scores over the international average, the United States falls behind Singapore, Malaysia, Latvia, and Bulgaria

1 The full 1995 and 1999 reports, as well as the 2003 report discussed below, can be accessed at http://timss.bc.edu/.

Among the many countries that outperformed the United States were Korea, the Slovak Republic, Hungary, Slovenia, the Russian Federation, the Czech Republic, Malaysia, Latvia, and my personal favorite, Bulgaria, which has a per-person economy that is only a small fraction of the U.S. per-person economy, yet nonetheless manages to educate its children better than we do here in the United States. Figure 3.1 compares the dominance of the United States from an economic perspective, based on 1999 World Bank figures,[2] to its lackluster performance on the education front, using TIMSS 1999 results. The comparison countries are Bulgaria, Latvia, Malaysia, and Singapore.

It gets worse. The U.S. eighth-graders described above performed far worse in comparison to students from other countries in the 1999 test than they did as fourth-graders in the original 1995 test. In other words, as noted by Dr. Gary Phillips, acting commissioner of education statistics, whose candor evidences a disconcerting lack of feel for the finer points of political office, "After the 4th grade, students in the United States fall behind their international peers as they pass through the school system."

Of course, all of the above is old news, based on data from a TIMSS study done in a prior millennium (i.e., from 1999). Have things improved since then? Fortunately, we have access to two major international studies, both released in late 2004: the next TIMSS report (TIMSS 2003) on the math and science performance of fourth and eighth graders; and the 2003 report of the Program for International Student Assessment (PISA 2003), which focused primarily on the mathematics literacy of fifteen-year-olds.

Let us begin with PISA 2003, which analyzes the math performance of fifteen-year-old students from the thirty countries that are members of the Organization for Economic Cooperation and Development (OECD). In essence, this study allows U.S. students to be compared with their age peers living in other economically developed countries. How did the U.S. fare in this comparison? The U.S. finished 24th out of 30 – i.e., in the bottom quartile![3] Here is a list of the countries that beat the U.S. in the 2003 PISA comparison:

Finland	Switzerland	France	Norway
Korea	New Zealand	Sweden	Luxembourg
Netherlands	Australia	Austria	Poland
Japan	Czech Republic	Germany	Hungary
Canada	Iceland	Ireland	Spain
Belgium	Denmark	Slovak Republic	

2 See http://devdata.worldbank.org/data-query/.

3 I say this secure in the knowledge that no one in the U.S. will be able to check my math!

Now here is a list of the countries that the United States beat in the 2003 PISA comparison:

Portugal
Italy
Greece
Turkey
Mexico

Do these results really need any comment?

How about TIMSS 2003 – is the news any better for fourth and eighth-graders? On the surface, yes, it is. The United States eighth-graders finished 15th out of 46 and the U.S. fourth-graders finished 12th out of 26, hardly spectacular results, but much better than the bottom twenty-fifth percentile.

But there is, as my grandmother used to say, a "flaw in the ointment." As noted above, the TIMSS study is heavily weighted with third-world and developing countries, countries such as Ghana, Indonesia, Botswana, Bahrain and the like. When the U.S. performance is compared with other industrialized countries in the TIMSS sample set, the U.S. fourth-graders finish right in the middle of the pack and the U.S. eighth-graders are slightly below average.[4] And, a cluster of Asian countries (most not included in the list of developed countries mentioned above) far outperformed the U.S. at both fourth and eighth grade levels. For example approximately one-third (33%) of the eighth-grade students in these Asian countries - Singapore, Chinese Taipei, Republic of Korea and Hong Kong SAR - reached the advanced TIMSS benchmark in math, while in the U.S. only 7% reached that benchmark!

Just for fun, we probably should take a quick look at the Department of Education's response to these two studies. On December 6, 2004, former Secretary of Education Rod Paige said, "The PISA results are a blinking warning light . . . It's more evidence that high standards and accountability for results are a good idea for all schools at all grade levels." [5]

A "blinking warning light"? Like the flashing red light that comes on when you have a telephone message? Twenty-fourth out of thirty, the bottom quartile, is not a "blinking warning light," it is a shrieking air raid siren; the bomb has fallen and the city is in ruins! On an IQ test, the bottom quartile corresponds to an IQ in the eighties!

4 See "Comparing NAEP, TIMSS, and PISA in Mathematics and Science," National Center for Education Statistics, U.S. Department of Education, http://nces.ed.gov/timss/pdf/naep_timss_pisa_comp.pdf.

5 Press Release, U.S. Department of Education, December 6, 2004.

What effect did the "blinking warning light" have on Secretary Paige's view of the TIMSS 2003 results? Eight days after issuing the above press release, the Department of Education reacted to TIMSS 2003 by stating, "America's fourth- and eighth-grade students significantly outperformed many of their international peers, scoring well above the international average in both mathematics and science . . . ." Of course, no mention is made of the fact that the countries the U.S. outperformed were the dregs of a stacked deck, countries such as the Philippines, Morocco, Chile, Egypt, Tunisia, Iran, Jordan, Lebanon, Cyprus, Ghana, Indonesia, Botswana, and Bahrain!

The release goes on to say, in a most disingenuous way, that "U.S. Secretary of Education Rod Paige expressed optimism with the TIMSS results, which measured students from up to 46 nations, including economically developed members of the [OECD]. . . ." Well, sure, the sample included OECD countries; the U.S. just did not outperform many of them! Yet, despite the stacked deck in the TIMSS 2003 study and the mediocrity of the U.S.'s performance against hand-picked competition, in only eight days Secretary Paige's blinking warning light has been supplanted by optimism.[6]

Why are our children so bad at math when compared with international children, and why does the gap worsen as they get older? It is not for a lack of effort! We have enacted the ambitious "No Child Left Behind" law, most states have established content standards and minimum course requirements in math and science, an increasing number of high-school students take advanced math and science courses, and in general, U.S. students receive as much instruction and do as much homework in these areas as students in other nations. So we are working a lot harder, but not necessarily a lot smarter. It appears that, as one distinguished professor of mathematics puts it, "we have not even grasped the true character of the problem."[7]

The problem is - astonishingly - evident even in physics graduate schools, where American graduate students routinely need remedial work on math concepts stretching all the way back to topics taught in high school. I saw this phenomenon first-hand, and believe me, it was embarrassing. The East Asian and European graduate students at the University of Virginia, where I received my physics Ph.D., were almost forced to conclude that Americans are just plain stupid, because the American graduate students could not seem to understand matters as simple (for physicists) as logarithms and trigonometric functions. It was particularly painful to watch the international students exchange knowing glances when an American students asked for a detailed explanation of a simple calculation - I felt like the relative of the guy who just fell off the turnip truck!

A number of theories have been advanced to explain the international math and science gulf.

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6 Press Release, U.S. Department of Education, December 14, 2004.

7 Lynn Arthur Steen, "Math Education At Risk," Issues In Science and Technology, Summer 2003

For example:

- We have the wrong people teaching science and math -- U.S. students are more likely than students in other nations to be taught by teachers with degrees in education, and less likely to be taught by teachers with degrees in math and science;
- We are not really working as hard as we say we are -- U.S. students devote a larger percentage of their classroom time to doing homework assignments; and
- We are teaching the wrong things -- the typical United States math curriculum focuses more on rote or repetitive skills and less on higher-level thinking skills.

I have my own theories about this, which I will lay out in the next chapter. But, to be honest, I am afraid that our efforts to improve math education in America are being guided too much by theory and not enough by common sense.

The situation is a little bit like that presented by the modern wave of diet books. We have the Pritikin Diet, the Zone Diet, the Atkins Diet, the South Beach Diet, the Scarsdale Diet, and so on and so forth, *ad infinitum* and *ad nauseum*. Each has a dramatic theory, typically involving a special diet rule that is claimed to be of overwhelming importance. Each has adherents who become almost disciples. And, in general each is inconsistent with the other diets in at least one important way.

This is a scenario that cries out for a little common sense. I suspect that you and I could write a very simple diet book that would lay out a diet program far superior to these fancy programs. Here is an example – we can call it the Four Rule Diet: (i) eat small portions; (ii) exercise forty-five minutes a day; (iii) eat mostly fruits and vegetables; and (iv) fill up the balance of your diet with chicken and fish. I know that we all love mysticism and intrigue; we all love to have access to that special non-intuitive insight that puts us in a special place of knowledge and power; but I will bet dollars to donuts that the Four Rule Diet is better and healthier than any of the above diets.

"We can continue to debate theories and tinker with various approaches, but while we are doing that, let us also do something that works; let us try the Pittsburgh Plan."

We are in the same boat when it comes to math education. We can continue to debate theories and tinker with various approaches, but while we are doing that, let us also do something that works; let us try the Pittsburgh Plan. The Plan is by its very nature flexible; it is in many ways agnostic about theory and happily heuristic in its approach. And children who go through the Plan have no trouble at all competing with international students!

CHAPTER 4:
The Mother Grizzly

"Is your children learning?"

--- **George W. Bush**, 2000

"It is nothing short of a miracle that modern methods of instruction have not yet entirely strangled the holy curiosity of inquiry."

--- **Albert Einstein**[1]

In the preceding chapters, we examined three separate problems with our American system of math and science education: the gender gap; the class chasm; and the international gulf. Perhaps as you plowed through these discouraging discussions, you began to get an uncomfortable feeling, the kind of feeling that a hiker gets when he encounters three small grizzly bear cubs on a remote trail – the prickly-neck, hair-standing-on-end feeling that comes when you just begin to understand the magnitude of your problem.

In this chapter, we will meet the mother grizzly; we will confront the terrible beast in its lair. And in doing so, we will come to understand that we do not have a series of separate problems, we have one huge, monolithic problem: the American system of math and science education is deeply broken. By every measure, our children are awful at math. They are awful compared to children in other countries, and they are awful compared to their parents and grandparents. Math anxiety is more common than math competence.

In previous chapters we discussed the poor math performance of American children in several different contexts. Believe it or not, though, we did not fully convey the magnitude of the problem. Our students' math performance is not just inferior when compared with the performance of foreign students; it completely fails to meet our own minimal standards of acceptability. For example, in the 2003 National Assessment of Educational Progress (NAEP), also known as the "nation's report card," 23% of fourth graders and 32% of eighth graders were below the "Basic" level of performance, while only 4% of fourth graders and 5% of eighth graders qualified as "Advanced" – this despite the fact that school systems have begun devoting large efforts to teaching directly to these tests to the exclusion or detriment of other material. [2]

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1 Quoted in H. Eves, Return to Mathematical Circles, Boston: Prindle, Weber and Schmidt, 1988

2 The NAEP can be accessed at http://nces.ed.gov/nationsreportcard. Bad as these statistics are, they do reflect minor improvement over the comparable figures for 2000, probably because of the "teaching to the test" effect mentioned above.

Former Secretary of Education Rod Paige reacted to these statistics by saying (I am not kidding, this is an actual quote), "We should be proud of our nation's teachers who led their students to these higher levels of achievement. They are the best in the world . . . . The first area to note is the stellar math results. Here we have very good news. . . . Kudos again to everyone—the students and their teachers—for these great marks." [3]

It is hard to fix a problem when you do not even recognize it! [4]

How can this be happening in the United States, the greatest experiment in free thought and rational inquiry ever conducted by human beings? Although no one knows for sure, it seems likely that that the problem results from a sort of resonance effect, where a number of different factors combine and reinforce each other. These factors include: (i) the self-fulfilling effects of the gender gap described in Chapter 1; (ii) the intractability of the class chasm described in Chapter 2; (iii) a "selection rule" that discourages people with significant technical talent from designing, creating, and teaching math and science curricula for elementary school students; and (iv) an unfortunate and growing tendency to place the blame for the problem on the children themselves. Put these factors together, stir and mix, and you have a prize-winning recipe for math anxiety and its faithful companion, math incompetence.

We have already discussed the first two factors at some length. It is probably worthwhile to give at least some thought to the other two factors.

The "Selection Rule." Think back to your school days. Remember the students who were good at math and science? Were they encouraged to become teachers or educational psychologists? Not after Sputnik and Tsar Bomba![5] Instead, it was assumed that they would become engineers or scientists. Education was a career choice suggested for children who did significantly better on their SAT Verbal tests than on their SAT Math tests.

This is what we mean by a "selection rule" (a term we have borrowed from particle physics). For a variety of reasons, people who are good at math and science are "selected" to pursue technical careers, meaning that the ranks of elementary school administrators, curriculum designers, and

3 Press Release, November 13, 2003, www.ed.gov.

4 Secretary Paige's successor, Secretary of Education Margaret Spellings, recently joined the Secretary of Education Glee Club when commenting on the 2004 NAEP report on long term trends, saying, "The results from the newest Report Card are in and the news is outstanding." Press Release, July 14, 2005, www.ed.gov. Outstanding? Although the report did show some improvement in math performance, especially at lower grades, it is likely that this improvement is attributable mostly to "teaching to the test" – a now-common practice in classrooms across the country. More importantly, the absolute performance levels of students, although improved in comparison with earlier years, remained dismal across the board.
See nces.ed.gov/nationsreportcard.

5 The Soviet Union launched Sputnik, the world's first artificial satellite, on October 4, 1957, and detonated Tsar Bomba, a huge hydrogen bomb, on October 30, 1961 on Novaya Zemlya Island in the Arctic Sea.

teachers tend to be filled primarily by other people – i.e., by people who are not necessarily expert at math and science. Of course there are exceptions, but the point is that on an overall statistical basis, people who excel at math and science tend to pursue engineering and the sciences instead of fields related to elementary education. The result? The people who design, create and teach the elementary school math curriculum – who are themselves often not "math-lovers" -- underestimate the math capability and math enthusiasm of the typical student.

The "Blame the Victim" Movement. When a problem proves to be particularly intractable, it is an unfortunate characteristic of human nature to blame someone who cannot defend himself. Thus you will perhaps not be surprised to learn that professors of psychology and education have recently discovered the existence of a math learning disability known as "dyscalculia," and have reluctantly begun to accept grants to investigate it further. (For some unknown reason, this condition apparently exists primarily in the United States.)

It is difficult to be sure of the extent of this disability, because researchers do not have a specific test for diagnosis. As a result, estimates about the frequency of occurrence of dyscalculia vary widely, ranging from my estimate of zero percent to estimates of up to eight percent by academicians who receive funding to investigate the phenomenon. One presumes that the same researchers are also hot on the trail of the Tooth Fairy!

It turns out that these dyscalculia experts are small potatoes when it comes to blaming the child for our American math education problem. After all, they only claim that some children are biologically wired to have trouble learning math. One noted expert, the chairman of the Department of Psychological Sciences at a major state university, trumps this with ease, saying that when it comes to math, "Much of what kids are expected to learn has been developed in the past 1,000 to 2,000 years, sometimes much more recently, and thus people's brains aren't really designed to learn much of it -- except, for instance basic counting and simple arithmetic .... Language, including foreign languages if they are introduced early enough, is a completely different matter, as the brain is pre-wired to learn this."[6]

Shades of Teen Talk Barbie! Math is hard! Perhaps the professor loves shopping?

The same article that quotes the above professor provides a refreshing example of a professor of education who actually seems willing to acknowledge that the problem may lie with the educational system, rather than with our children. Professor Art Baroody, of the University of Illinois at Urbana-Champaign, states that he is "willing to bet the ranch" that the primary culprit for students' math struggles is poor instruction, and goes on to say, "Children with genuine organic dysfunction probably make up a small portion of the children struggling with math, or even of those labeled

6 Quoted in Valerie Strauss, "Trying to Figure Out Why Math Is So Hard for Some," Washington Post, December 2, 2003, accessible at : www.washingtonpost.com/ac2/wp-dyn?pagename=article&node=&contentId=A26826-2003Dec1&notFound=true.

'learning disabled'. . . It is probably never too late [to learn math], if the spirit has not been broken."

How did they ever let this guy into the club?

Putting all of these various factors together, we can see that the typical young child is immersed in an environment telling her that math is hard, that only certain people have math ability, and that it is normal to be afraid of math and not very good at it. This view – in essence, a certain type of fear of math -- permeates the home environment, the media, and even the schools. Unfortunately, that fear is probably the single thing that gets transmitted most effectively to young children. How does it get transmitted?

First, by parents, media characters, and teachers, through what they say and do in the presence of the child. I have often heard parents and elementary school teachers say things like, "I am not a math person" and "Yikes, I cannot do fractions!" Such statements are usually made laughingly, with a sort of apologetic, self-deprecatory shrug. Now, there is no point in blaming these people for their math inadequacies, because they are products of a system that is in shambles. But that does not mean that this kind of attitude is acceptable. How would we feel if a parent or teacher were to say, "I cannot read two-syllable words" or "I am not a reading person"? Would we not be deeply troubled by that illiteracy? Why is math illiteracy any different? And why do we as a society communicate our fear of math, and our math illiteracy, to students as if it were some sort of silly little foible, like being a terrible cook or having trouble doing somersaults?

". . . the typical young child is immersed in an environment telling her that math is hard, that only certain people have math ability, and that it is normal to be afraid of math . . ."

Second, this fear of math gets transmitted by the textbooks. Just as an example, I have pulled out of my bookshelves an eighth-grade math text entitled Integrated Mathematics 2, McDougal Littell (1995). I believe that this book was in use by my local school district until September of 2006.

Now, if you were to look at an eighth-grade history text, the content would be pretty much one hundred percent history. Likewise with other subjects such as literature, French, and English – each text would focus almost exclusively on the course's subject matter. You can be quite certain that none of these texts would make any attempt at all to draw any connections to math!

". . . why do we as a society communicate our fear of math, and our math illiteracy, to students as if it were some sort of silly little foible, like being a terrible cook or having trouble doing somersaults?"

Things are very different with the math textbook. When you open Integrated Mathematics 2, you are first struck by the fact that the pages are full of color photographs having nothing to do with math – pictures of musicians, athletes, and almost everything else under the sun (see Figure 4.3).

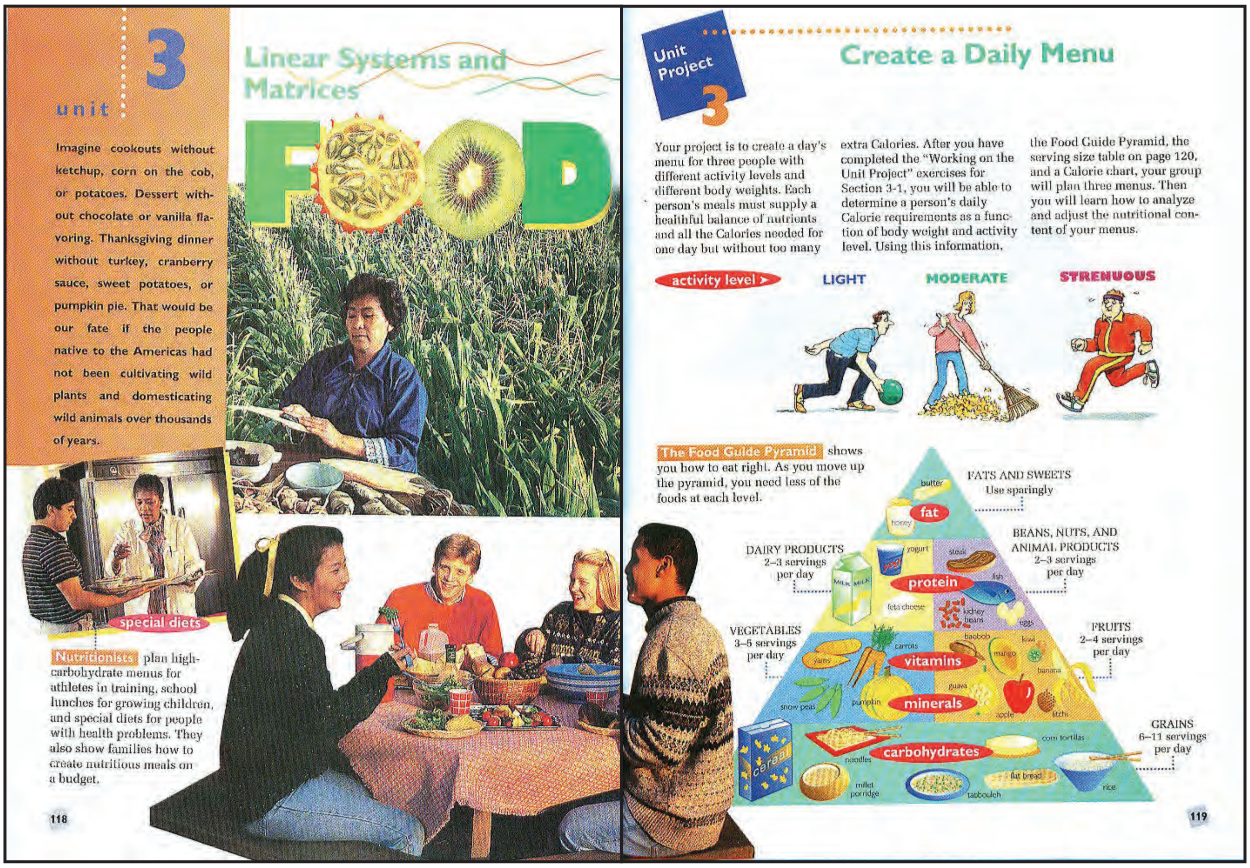


FIGURE 4.3: Two sample pages from Integrated Mathematics 2, McDougal Littell (1995).

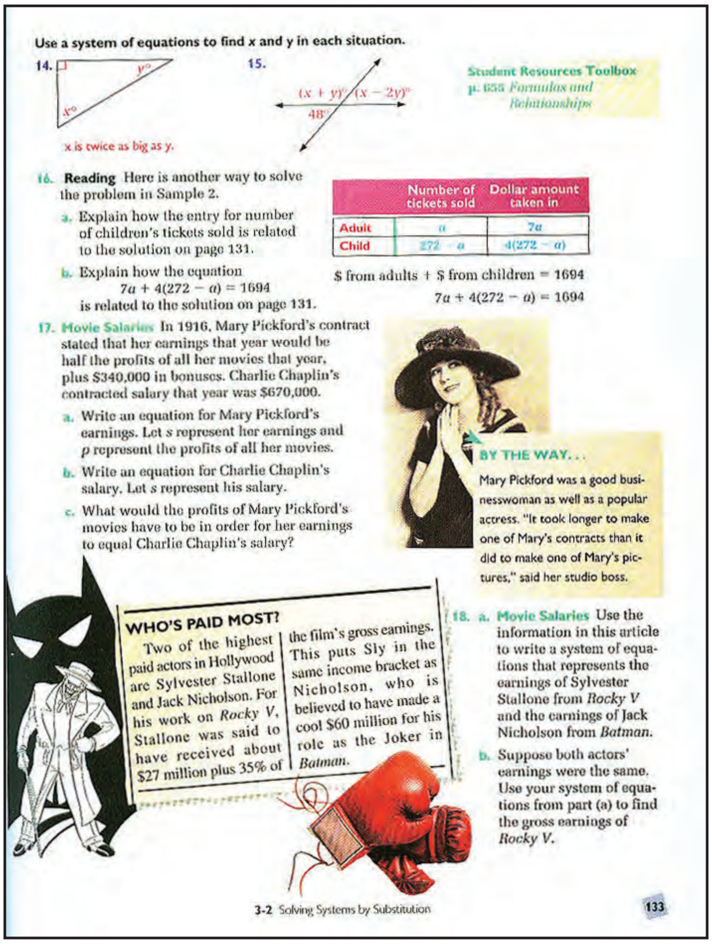
Then you notice that it is actually a little hard to find the math content, because there are so many "interesting asides" having nothing to do with math. Every Unit has a unit project –making a daily menu, for example -- that involves a small application of math in the context of much more material about some other topic, e.g., nutrition, in the case of the menu project. The pages are full of color and are artfully designed, so much so that once again the math content is almost concealed. In fact, the book has the overall look and feel that one would expect of a third-grade text.

Perhaps most significantly, Integrated Mathematics 2 makes overwhelming efforts to present material other than mathematics. Since we are not afraid of math, let us quantify this. Drawing from the index of Integrated Mathematics 2, we see the following, all in addition to the many pretty pictures and child-like graphics:

- 30 "By the Way" discussions of things unrelated to math (see Figure 4.4 below);
- 10 "Can We Talk" discussions of things unrelated to math;
- 33 "Discussions" of "History" (non-mathematical);
- 12 "Discussions" of math history;

- 13 significant excerpts from literature (fiction);
- 47 "Connections" discussions of other disciplines;
- 16 "Discussions" of careers;
- 64 "Applications" to other fields, each involving significant discussion of the other field;
- 130 "Multicultural Connections" discussions;
- 21 projects, many involving only incidental math; and
- 10 large unit projects, each involving relatively minor math applications in the context of large amounts of non-math material.

The authors of Integrated Mathematics 2 are so convinced that math is boring that they do everything but pay cash bribes to entice the students to read a little bit of math material! It would never even cross their minds that pure math – algebra, geometry, probability, etc – is every bit as interesting as history or literature. These guys do not love math, they are afraid of it. And as a result they just cannot teach it.



Use a system of equations to find x and y in each situation.

14. x is twice as big as y.

15.

Student Resources Toolbox
p. 655 *Formulas and Relationships*

16. **Reading** Here is another way to solve the problem in Sample 2.
  a. Explain how the entry for number of children's tickets sold is related to the solution on page 131.
  b. Explain how the equation
  $7a + 4(272 - a) = 1694$
  is related to the solution on page 131.

	Number of tickets sold	Dollar amount taken in
Adult	a	$7a$
Child	$272 - a$	$4(272 - a)$

\$ from adults + \$ from children = 1694
$7a + 4(272 - a) = 1694$

17. **Movie Salaries** In 1916, Mary Pickford's contract stated that her earnings that year would be half the profits of all her movies that year, plus \$340,000 in bonuses. Charlie Chaplin's contracted salary that year was \$670,000.
  a. Write an equation for Mary Pickford's earnings. Let s represent her earnings and p represent the profits of all her movies.
  b. Write an equation for Charlie Chaplin's salary. Let s represent his salary.
  c. What would the profits of Mary Pickford's movies have to be in order for her earnings to equal Charlie Chaplin's salary?

BY THE WAY...
Mary Pickford was a good businesswoman as well as a popular actress. "It took longer to make one of Mary's contracts than it did to make one of Mary's pictures," said her studio boss.

WHO'S PAID MOST?
Two of the highest paid actors in Hollywood are Sylvester Stallone and Jack Nicholson. For his work on *Rocky V*, Stallone was said to have received about \$27 million plus 35% of the film's gross earnings. This puts Sly in the same income bracket as Nicholson, who is believed to have made a cool \$60 million for his role as the Joker in *Batman*.

18. a. **Movie Salaries** Use the information in this article to write a system of equations that represents the earnings of Sylvester Stallone from *Rocky V* and the earnings of Jack Nicholson from *Batman*.
  b. Suppose both actors' earnings were the same. Use your system of equations from part (a) to find the gross earnings of *Rocky V*.

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FIGURE 4.4: Another page from Integrated Mathematics 2, McDougal Littell (1995). Note the rigorous treatment of Mary Pickford and Hollywood actors' salaries.

It is not hard to see that we have created a vicious cycle of math-phobic adults, who beget and teach math-phobic students, who become the next generation of math-phobic adults, and so on.

So, our society is broken, at least when it comes to teaching math and science. How do we fix the problem? In the short run, I do not have an answer. All I can tell you is that you can probably help your young child dodge the worst effects of this problem by adopting and using the Pittsburgh Plan.

Think of the Plan as a vaccination, an inoculation protecting your child against the epidemic we call American math education. Who knows? Perhaps once we have vaccinated enough children, the epidemic will resolve itself. Perhaps someday we will have a new generation of parents, curriculum designers and elementary school teachers who love and embrace math, rather than fearing it. And perhaps someday pigs will fly.

Chapter 5:
So Who Is to Blame?

"I was run over by the truth one day . . . ."

--- **Adrian Mitchell,** "To Whom It May Concern," 1964

"If you want to end war and stuff, you got to sing LOUD."

--- **Arlo Guthrie,** "Alice's Restaurant," 1966

Now that we have four chapters' worth of background information, we are ready to ask the question, "Who is to blame for our children's failure to learn math?" Does the fault lie primarily with our parents? Our teachers? With the educators and psychologists who design our curricula? Or, as some of these people would assert, with our children themselves?

As I began my quest, years ago, for a decent education for my children, I found myself blaming almost everyone: the teachers who insisted on spending a week on every letter of the alphabet ("Now we will learn about 'Mr. Q!'") with children who could read Charlotte's Web; the administrators who refused even to consider testing children for placement in math levels above their grade; the curriculum designers who decided that tessellations should be taught (over and over again) instead of multiplication and division . . . . And then I had an insight; I was run over by the truth. It does not matter who is to blame! In the end, as parents, we are responsible for our children. If our children end up being poor readers, we can blame our school systems until we are blue in our faces – and possibly even be right in blaming them! – but our children will still be poor readers. If our daughters cannot do math, does it really matter that we can righteously and wrathfully place the blame on someone else? Our daughters will still suffer and live lesser lives than they otherwise could.

"It is a cold, cruel world, and there are no points awarded for being a victim. So, who is to blame if our children cannot read or do math? We are!"

It is a cold, cruel world, and there are no points awarded for being a victim. So, who is to blame if our children cannot read or do math? We are! It does not matter if it is fair, or just, or right; the hard truth is that the fate of our children rests ultimately on our own shoulders. And, in the end, it is our job to make sure that our children receive the education they need. We have no choice; we must do whatever it takes.

So, when we encounter teachers who are uncomfortable with math, or administrators who seem more concerned with protecting the integrity of their curriculum than with the actual effectiveness of that curriculum, we have to take a step back and think constructively. Of course we are irritated, of course we want to blame them, but that does not get us anywhere. Instead, we need to solve the problem of educating our children.

How do we do this? Obviously, it would be best if we could fix the system that is broken, so that we solve the problem for everyone, not just for ourselves. If you try to do this, you will discover, as did I, that there are vast forces opposing any systemic change. The gender stereotype, the factors leading to the class chasm, the math anxiety that pops up in many homes and classrooms, the pernicious notion that many children simply are not "math people" – these influences are by now deeply entrenched in our society, and lend each other mutual support against any frontal assault.

So, we need a Plan B. Plan A will not work; we will not be able to fix the system, at least not by ourselves, and certainly not in time to save our own children. But we can take steps to save our children; we can take responsibility for their futures. And, if enough of us do that, if enough of us refuse to accept the limits our society seeks to place on our children, pretty soon we will have a movement. Then, after a while, our movement will become an army. An army of children that can do math! And then some of them will become parents and teachers, and a brave few will travel into the land of Canaan and become educational psychologists, and then we will have broken the self-perpetuating cycle of math anxiety and underperformance.

But it all starts with you -- with you making sure that your own child has the opportunity to reach her potential. And the best way that we know to do this is for you to follow the Pittsburgh Plan, to spend fifteen minutes a day, a few days a week, working with your child. Perhaps we will never attain critical mass to make a difference in American society as a whole, perhaps we are too few ever to overcome the formidable forces arrayed against any such effort, but in that case, at least your child will be an island of competence in a sea of ignorance, free to achieve her own hopes and dreams. And to the extent American society needs large numbers of people with ability in math and science, we can always continue to recruit them through free agency from China, India and the rest of the world.[1]

> **". . . It all starts with you -- with you making sure that your own child has the opportunity to reach her potential."**

Now, pause for a minute and think back upon the original objectives of the Pittsburgh Plan. Perhaps you recall from the Introduction that our objectives were to (i) foster your child's innate love of learning, (ii) enable your child to reach the level of academic mastery that her abilities will permit, free of obstruction, and (iii) develop in your child a justified sense of self-esteem that will help carry

1 We are all proud that America is the "land of opportunity." But have you ever stopped to wonder exactly why the rest of the world views us that way?

her through the many challenges of life. Note that there is nothing in these objectives that has anything to do with the pervasive math under-performance in our society and our schools!

The Pittsburgh Plan is not designed to fix the problems of society or our school systems. It is not really intended to fill gaps left by our inadequate curricula. In fact, the Pittsburgh Plan works even better when paired with a great and flexible school system. The Plan is a pre-school and out side-of-school program; it exists independent of the school system, and has different aims. It does something that no school system anywhere has ever envisioned – it allows children to learn whatever they are capable of learning, whatever they want to learn, without artificial obstacles imposed due to preconceptions about the limits of a child's potential.

In a sense it is an accident that the Pittsburgh Plan is an effective antidote to our math problems in America. That effect is merely one manifestation of a much larger, more important effect; by harnessing the intellectual power of our children at early ages, we can help them achieve great things. We can help them live lives of excitement, success and happiness. These are the real reasons to try the Pittsburgh Plan!

Chapters 6 – 8 lay out the core principles of the Pittsburgh Plan. Armed with these basic guidelines, you may, if you wish, proceed directly to Appendix A and begin using the "canned" plan with your child. Alternatively, you may wish to continue on to Part 3: "Nuts and Bolts of the Pittsburgh Plan," either before or concurrently with your commencement of homework sessions.

Chapter 6: *Two Cornerstone Assumptions*

In my early days as an attorney, I did a great deal of work in the booming real estate market of California. Fortunes were made overnight as speculators bought properties on credit, banking on the twin growth engines of high leverage and double digit inflation. It all seemed to be too good to be true.

And it was! Suddenly rates skyrocketed, mortgages became very expensive, and buyers disappeared. And almost before one could blink, all the high-flying, highly leveraged developers were driving BMWs with holes in the trunk where the cell phone antennae used to be. And a few weeks later, they were driving used Dodge Darts.[1]

I remember wondering how a whole market – really, a whole society -- could have forgotten Polonius' famous admonishment to his son Laertes, "Neither a borrower nor a junior lender be."[2] Sure, Polonius was a blowhard, but part of the charm of that passage is that, however galling his pontifications may have been, his advice was solid; everyone knows this rule, and everyone knows that it is a good rule. So why did Newport Beach suddenly become the Dodge Dart capital of the world? How could thousands of near-sentient individuals ignore this rule? Probably the answer lies in another old adage, "Familiarity breeds contempt." The rule was too familiar; it became obscured by its very ubiquity.

The lesson here is simple: we should not ignore ideas simply because they are familiar. The problem with most clichés is not that they are false; it is that we do not pay attention to them because they are so familiar.

The Pittsburgh Plan applies this lesson with a vengeance. The Plan was developed based upon two simple assumptions: (i) every child is unique; and (ii) a child's best teachers are her parents. There is nothing novel about these ideas; in fact, they are so commonplace that they have become clichés, trite phrases without much content.

But, we can profit from the experiences of our insolvent developer friends in California; we can give these ideas a fresh look and a fair hearing. Why? Because these two simple assumptions do in fact have important content, and they give us a good deal of guidance in fashioning an approach to teaching our children. You will see that the Pittsburgh Plan is logically connected to these important ideas, and that they help us understand how best to structure our homework sessions with our children.

Every child is unique. Certainly we all accept this and pay lip service to it, but how do we act on it? How do we design a child's learning environment to take account of it? What does this mean about the ways in which a child should be taught – or, more broadly, the ways in which a child should be given an opportunity to learn?

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1 Apparently cell phone repossessors work a little faster than automobile repossessors.

2 Well, this is what Polonius would have said had he lived in Orange County, California in the early 1980s. See Hamlet, Act I Scene III.

One obvious consequence of this assumption of uniqueness is that children will have different strengths and weaknesses. Some will be stronger or faster than others; some will be better at math or reading than others. But this is a pretty narrow reading of the idea of uniqueness.

Anyone who studies cognition and intelligence quickly comes to realize another very important aspect of this general idea of uniqueness: every person has different preferred ways of learning and thinking. Some people learn best by reading; others by hearing lectures. Some people think best about math problems using spatial reasoning or intuition; others prefer to employ logical skills. Some people solve problems or tasks by grinding forward one step at a time; others just toss the whole problem into a sort of mental Cuisinart and let things blend for a while until a solution emerges. Some people think best while seated quietly and concentrating intensely; others prefer to move around in an environment full of background noise and activity.

In fact, it is safe to say that we do not understand intelligence, learning and thinking very well at all, and that there are probably many strategies, abilities, techniques, and preferences that we cannot even identify. All that we can really be sure of is that different children need different learning environments.

What does this mean for us? What does this tell us about teaching our children?

Well, first, it suggests that we should not try to force a particular method of learning or thinking upon a child. We should instead present the child with an environment that is rich in content, and then allow the child to determine, with our help, how to deal with that content. In other words, we allow the child to pick her learning strategies, although we make a strong effort to help her get exposure to as many different possible approaches as possible. Stated yet another way, we teach interactively rather than didactically; we listen as well as talk.[3] For logistical reasons, this is simply not possible at an elementary school, but you will see that the Pittsburgh Plan adopts this approach whole-heartedly.

> **"we teach interactively rather than didactically; we listen as well as talk."**

So that we remember this important idea, let us state it as a guiding principle. Because the real core of this idea is that we are paying attention to our children and their own particular strengths, we call this "respecting the child":

Guiding Principle #1: Respect Your Child: Do not tell your child the "right" way to approach a homework task. Let her struggle with it for a while, and perhaps develop her own approach or strategy. Ask your child what she is thinking, and listen to her thoughts whenever she offers them. When you provide aid, do so as a partner rather than a teacher; provide alternative strategies, and point out that your suggestion is just one of many possible approaches.

3 Listening to your child is so important that we state it as a priority no less than eight different times in this book!

It is worth noting in passing that this interactive approach is very much how children learn to speak their native language – they are immersed in it and figure it out without a great deal of formal instruction. We will return to this important idea of native language acquisition later.

The idea that every child is unique also leads us directly to the second assumption, that <u>a child's best teachers are her parents.</u> Why? Because if anyone knows a child well enough to have a sense for her preferred learning strategies and cognitive strengths, it is a parent. If anyone is going to take the time to tailor a learning environment specifically for a child's needs and strengths, it is a parent. This does not reduce the importance of a flexible, non-didactic learning environment, it merely suggests that whatever structuring inevitably occurs is best done by a parent.

There are other, even more important reasons why a child's best teachers are her parents. Learning occurs best when it is fun, and for young children there are no better times than times with parents. Young children are more engaged, more active, and more involved in an activity when it is part of a parent-child interaction. Also, children love to demonstrate proficiency to their parents and receive praise for it; this greatly facilitates learning. Finally, the most important operational principle of the Pittsburgh Plan is that every session should be fun and free of pressure, and no one is more likely to have the necessary patience for this purpose than a parent.

So, if a parent is the best teacher, what does this mean for our design of an instructional program? It seems pretty obvious, does it not? The parents should be the teachers! This is not possible for an elementary school, but it certainly is for our program, and it is very much a part of the Pittsburgh Plan. So, we have our second guiding principle:

<u>**Guiding Principle #2:** The Homework Sessions Should Be Conducted By a Parent</u>: Here, when we say "parent," we really mean "primary caregiver" – the person whose opinion the child most values, whose approval she most craves, and whose companionship she most enjoys.

It is surprising how often parents groan at this point. This is not a bad thing or a disadvantage; it is the great equalizer. Perhaps you cannot afford to send your children to the best private schools, or hire special tutors for them, or send them to expensive academic camps. It does not matter! You can do something that trumps all of that. You can spend fifteen minutes a day, four days a week, with your child. And if you do, the odds are that your child will outperform the rich kid who has every advantage except that kind of quality learning time with her parents.

This sounds a little far-fetched, doesn't it? Almost as far-fetched as the idea that two African-American sisters could learn tennis from their father on public courts, without privileges or amenities, and someday become the best tennis players in the world. One sister might be an accident, but two sisters is a result. Richard Williams did something right, and I suspect that what he did was spend time with his daughters.

For reasons that are not entirely clear, parents are very good at making time for their children when it comes to producing good lacrosse players, baseball players, swimmers, pianists, tennis players, and the like, but have much more trouble making time to help their children excel in fundamental cognitive skills such as reading and math. In fact, parents are sometimes reluctant to adopt the Pittsburgh Plan because they fear that it might require their child to drop one of several sports in which the child is formally engaged. They are afraid that eight or ten years down the road some other child will make the wrestling team, or the football team, or the volleyball team, instead of their child. This seems a bit far-fetched! But even if these parents are right, and Johnny might be risking his varsity letter in wrestling by devoting fifteen minutes a day four times a week to reading and math, it is important to recognize that there is a pretty important quid pro quo! Johnny may not be a varsity wrestler, but he will be a lot smarter than he would have been absent the program.

We have been approached many times by people who have sought to partner with us and invest money in the commercialization of the Pittsburgh Plan. Often, they view it as a perfect Internet-based business, where worksheets can be put on the Web and accessed by computer. In fact, it would be fairly easy to develop expert systems that would grade worksheets and intelligently assign the next days' homework. We have always resisted this approach, because we are afraid that we know how it would end up: with parents plopping their kids in front of the computer screen while the parents do other things. And you can see why that idea is not palatable – after all, it violates one of the two cornerstone assumptions underlying the whole Plan!

The Internet-based idea does make sense as a convenience for the parent, and it may be that someday we will go ahead and pursue the Internet path for that reason, but only if we can figure out a way to do it that ensures continued active parental involvement.

We should probably talk for a minute about home-schooling, which after all has a strong connection to the idea that a child's best teachers are her parents. If the Pittsburgh Plan is a logical consequence of this idea, should we not take things to the next level – i.e., full-scale home schooling -- and make the entire day of teaching more effective, instead of just trying to salvage an occasional fifteen minute interlude?

The problem with this idea is that, unlike the Pittsburgh Plan, home schooling has a number of significant downsides, including the lack of normal social interactions, difficulty in getting access to extracurricular activities, restricted exposure to broader views, the loss of an intermediate step between an environment dominated by loving and nurturing parents and the completely self-dependent college environment, large extra expense, and a huge commitment of parental time. Also, we know from experience how hard it is to design a curriculum in just one area in which we are expert (math), and fear that it would be very difficult for a small group of parents to do a good job in developing and presenting an entire curriculum, inevitably involving areas where they lack real knowledge.

Finally, elementary school teachers are a dedicated and impressive group of people, and we fear that children might suffer from the loss of exposure to them.

On the other hand, although we admire teachers, we have come to distrust the educators who design the public school math curriculum. That curriculum is so poorly conceived that it can fairly be called incompetent. It is true that a great deal of time and money went into the development of modern math curricula and texts, and a large number of people behind them have impressive credentials. On the other hand, the same could be said of the Edsel and New Coke! Anyway, a lemon is a lemon, and it is certainly possible that the curriculum as a whole is as flawed as the math curriculum – to be honest, we do not know, because we have not taken the time to examine it critically. So, we can understand why parents might feel that home schooling is an imperative.

On balance, we have concluded for our own children that the many benefits of regular school outweigh any academic detriments, particularly if the latter are addressed through the Pittsburgh Plan. But, the Pittsburgh Plan does not really weigh in on this debate; the Plan is designed for use as a supplement to a regular school education, but could also easily be used as part of a home schooling program. It is important to note, though, that the Pittsburgh Plan assumes that a significant amount of rote math learning ("math facts") is occurring elsewhere, and any home schooling use of the Pittsburgh Plan should supplement it with such rote material.

Chapter 7:
The Path is the Destination

Once upon a time . . . .

The standard American family consisted of a husband who worked forty hours per week, a "non-working" wife who was responsible for the household,[1] and children who were free to play most of the day but knew there was a parent nearby if needed. There was sufficient money for basic needs and a sense of optimism that advances in technology would soon bring both prosperity and a reduced work week.

Perhaps life was never really this simple. Certainly, major problems lurked beneath the surface of this happy picture, notably the many obstacles confronting women who sought a different path and the monochromatic nature of the American middle class. But, for the families that lived it, this lifestyle seemed to be a good one for children, providing them with a solid emotional base and a strong sense of security.

Things are very different now. Contrary to mid-century expectations, the technological revolution of the past fifty years has not reduced the pressure on the average person's time. In fact, the notion that technology would lead to four-day work weeks and an easing of economic burdens has proven to be entirely wrong. Today, for a wide variety of reasons, many families have two working spouses, each working well over forty hours per week. Even for families that have a stay-at-home spouse, the pace of life has changed dramatically. The traditional American family is largely a thing of the past.

An obvious consequence of the change in the American family is a reduction in the amount of time that parents spend with their children. Once a young child would spend almost all of her waking hours in the presence of a parent; now it is not unusual for a child to see her parents only in the morning and the evening, before and after work. This has led to great pressure on parents to make their time with their children count extra – i.e., to counter the reduction in the quantity of their time with their children by an increase in the quality of that time.

This sounds sensible, doesn't it? If we cannot give our children as much of our time as we would like, we will instead make sure that our time with them is maximized, so that they are not cheated. Of course, this is praiseworthy. But it does raise two very important issues.

First, can quality of time truly be a substitute for quantity of time with a child? Can a parent make up for a reduction in his time with his child by increasing the quality of the remaining time they have together?

1 Anyone who has done this job will understand why we put the term "non-working" in quotes!

It is certainly conceivable that in the case of a young child there is no adequate substitute for the all-day presence of a parent. This is a very uncomfortable thought! Because, if it is true, it places great pressure on parents to give up their careers and stay at home with their children. But, the fact that this idea is uncomfortable does not mean that it is wrong, or that we should ignore it or assume it away.[2]

Do children benefit significantly from having a stay-at-home parent? A complete analysis of this question is beyond the scope of this book. After all, we are focusing on how to approach fifteen-minute sessions with our children, not on major life choices. But this issue does in some sense inform our approach to our fifteen-minute sessions. The Pittsburgh Plan takes the view that quantity of time is important; that relaxed and happy time with a parent has significant benefits for a child, regardless of whether that time is used in a highly "effective" manner. Thus, even if the entire theory behind the Pittsburgh Plan is incorrect, and the initial successes of its novel approach to math learning are merely coincidences, the Plan will benefit children by giving them an extra hour each week with a parent.

Our most broad-brush view of a homework session under the Pittsburgh Plan is that it is merely a period of relaxed parent-child time. We are not seeking greatness in each session; we are content at the most basic level just to add another fifteen minutes to the child's comfortable, happy time with her parents. So, perhaps the best simple description of a fifteen minute homework session is that it is "some time together."

First Definition of the Homework Sessions: *Some time together.*

The second issue raised by the idea of "quality time" is fairly simple. How do we measure quality? What are the characteristics of high-quality time between a parent and a child? Although this is a question of general importance, the Pittsburgh Plan takes a particular interest in it, because we are allotting to ourselves only one hour per week.

Perhaps it is best to begin by examining what is not quality time.

We noted above that technological advances have not had the once-expected effects of reducing the work week and easing economic pressure on middle class families. Instead, the opposite seems to have happened. We spend more time at work and have less time for family; at the same time, we are experiencing ever-increasing economic stress.

Meanwhile, as the economic value of our time has increased, we have come to feel great pressure to use that time efficiently. We work long days, eat on the run, drive fast, and sleep less than we

2 Logically, there is no reason why pressure to be a stay-at-home parent should not fall equally on husband and wife. However, it is likely that any such pressure will fall primarily on women, who will be asked to surrender their hard-earned social gains and resume their historical roles as stay-at-home caregivers.

should. Our pace is fast, we multi-task, and leisure time has become at best an expensive luxury. We even schedule time with our children to make sure that we do not miss them entirely during the day! We do not necessarily like this roller-coaster existence, but we adapt because we must; we approach time like efficiency experts.

One of the unfortunate side effects of this fast-paced lifestyle is that we have become confused about the idea of quality time for children. We think about our children's schedules in the same way that we think about our own; we schedule their time efficiently to maximize the value of their time. Does a young boy like music? We enroll him in Suzuki piano lessons and modern dance lessons. Is a young girl fleet of foot? We sign her up for multiple soccer leagues and send her to weekend track clinics.

In other words, because time with our children has become so precious, we naturally feel pressure to make the most of that time. As a result, we schedule our children's time – all of their time, not just their time with us -- as rigorously and carefully as we do our own. And in doing this, we place great pressure on all aspects of our children's lives: sports; music; school; and everything else.

" . . . because time with our children has become so precious, we naturally feel pressure to make the most of that time."

The problem is most obvious at its extremes, when parents berate young children for athletic "failures" -- failing to block a shot or catch a pass -- and badger officials who make routine calls adverse to their children. We have all seen this or read about it, and we deplore it as being inappropriate. But have you ever looked carefully at the child when this kind of behavior is going on? Her shoulders will droop; she will get a distant, uncomfortable look; sometimes you can almost see her withdraw into herself. An activity that was once fun has instead become just one more area where she is somehow not quite measuring up to her parents' expectations; softball, or soccer, or football, is no longer fun, instead it is a scary proving ground where a child must earn her parent's approval.

There are many reasons why this kind of parent-child dynamic is a bad thing. For example, it places far too much emphasis on building a child's skills, and not enough on building a child's character. By substituting instruction for play, we produce a young person who can kick a soccer ball but does not know how to make a friend, who can run fast but cannot share a toy.

Often parents respond to this criticism by pointing out that overcoming adversity in sports builds character, but this is a very limited view of character, emphasizing those attributes that enable successful competition. Certainly one must compete well to enjoy a fully productive life, but one must also learn to have happy and productive relationships with friends, a spouse, and children. And, ideally, a person should be able to appreciate and enjoy his or her daily activities for their intrinsic worth, not just for the results they produce (e.g., a paycheck). These are elements of character that cannot be forged in the crucible of intense competitive pressure, but instead must emerge from a more relaxed and happy environment.

It would be fair for you to ask at this point, "Just what does this all have to do with the Pittsburgh Plan?" We have three answers, each tying the above discussion to our approach to the fifteen minute homework sessions.

First, at the most general level, we like the idea that there is no downside to the Pittsburgh Plan; that it involves no expense and requires only a small increase in the time a parent spends with his child. The Plan will have a positive effect at this most basic, minimal level so long as the parent-child time is a good thing and not a bad thing; i.e., so long as we do not burden these sessions with anxiety-producing parental pressures and expectations.

Second, despite our concern about avoiding downside, we actually have high expectations; we believe that the Plan will help your child read early and develop great mathematical skills. In order to achieve these extraordinary results, the Plan depends heavily on harnessing your child's innate language-learning ability. Our experience indicates strongly that this powerful learning machinery operates far better in an environment of happiness and enthusiasm than in a climate of pressure and anxiety.

Third, our objectives are not limited to academic achievement. Obviously, we feel that early reading and strong math proficiency are good things, but in a sense they are only means to an end. We have two much larger objectives: (i) we want our children to develop a strong sense of <u>earned</u> self-esteem, a sense of capability that will allow them to continue to approach challenges with confidence and determination throughout their lives; and (ii) we want our children to preserve their sense of wonder and joy at learning throughout their lives, so that their lives are richer and more rewarding even as adults. Both of these objectives – self-esteem and enjoyment of learning – suggest strongly that the homework sessions should be fun and free of anxiety and pressure.

These ideas are important. Let us explore them a little more deeply, and perhaps refine our definition of a homework session to accommodate them.

One of the nice features of the Pittsburgh Plan is that it has no real downside. Even if the underlying theory is all wet, the worst-case scenario is that you will be spending an extra fifteen minutes each day with your child. So, there is really nothing to lose in trying the Plan, right?

This is true, except . . . . there is one way that the program can have a negative effect. Occasionally, a parent can become frustrated with his child's apparent lack of progress, and this can lead to pressure on the child. The parent may exhibit impatience, or even suggest that the child is slow, or not trying. This defeats the whole purpose of the Plan.

For example, in the Pittsburgh Plan we teach addition with carrying (sometimes called "regrouping") very early. An example would be 15 + 16, where the child must add 6 + 5, getting 11, and then "carry" a 1 to the tens column. One of my young children had mastered this technique and had used it flawlessly in eleven consecutive homework sessions, but for some reason in the twelfth session took a step backward and could not do this kind of problem. I found myself getting impatient and a little frustrated . . . then suddenly I looked at my child and realized that she was visibly shrinking into herself. I had somehow communicated my unhappiness to her, and she was very sad.

Needless to say, this changed my attitude immediately; I retrenched, apologized, and moved on to other types of problems. Things worked out fine; my child was happy in subsequent sessions and had no trouble working out carrying problems. I still do not know exactly why she had that one-day problem, but I do know that such things happen all the time. We are dealing with children, not little adults, and they think and behave differently. In this case, the child was only four years old; I should have been thrilled that she was doing addition with carrying at all, rather than bothered by her momentary step backward.

"We are dealing with children, not little adults, and they think and behave differently."

Let me give you another example. Young children frequently spend many months being able to sound out the components of words but not being able to combine the sounds into full words. A child may be able to sound out "h," "a," and "t," but may not be able to put those sounds together to identify the word as being "hat." And this lack of phonetic integration can go on for months.

It is natural in such cases for the parent to feel that the child is not making progress, or is not making an effort. If this concern is communicated to the child, even subtly and indirectly, the happy times together may be replaced by tension and anxiety. Learning will no longer be fun, it will be a formidable and daunting task. And the Pittsburgh Plan will not work, because the child will disengage emotionally, and her great innate learning mechanisms will no longer be brought to bear in the sessions.

We want to avoid this. So, although we have larger goals, as discussed above, we adopt as an important initial precept the physician's canon: "First, do no harm." [3] In our case, this means that we must keep the fifteen minute sessions fun and free of pressure. This leads us to refine our definition of a homework session to be, not just "some time together," but instead "some relaxed, happy time together."

3 Contrary to popular belief, this phrase is not part of the Hippocratic Oath. It may have originated in another of Hippocrates' writings, Epidemics, Bk. I, Sect. XI, which (translated) reads: "Declare the past, diagnose the present, foretell the future . . . . As to diseases, make a habit of two things—to help, or at least to do no harm." Alternatively, it may have originated with the Roman physician Galen.

<u>Second Definition of the Homework Sessions:</u> *Some relaxed, happy time together.*

We have come a long way from our original definition of a homework session, expanding it by almost seventy percent.[4] And, so far, all of our refinement has come from considering what is <u>not</u> "quality time." Let us push things a little further by thinking about the kinds of things that might <u>be</u> "quality time," at least in the context of a homework session.

The right way to approach this question is to come at it from the perspective of the child: **"What kind of time does a child need from her parents?"** In particular, how can we create conditions that help the child learn, that take advantage of the power of the child's innate language ability? And what can we do to help achieve the key objectives of the homework sessions: to build competence and self-esteem; to encourage and maintain a love of learning?

Children do not just love to learn, they bring to the task sophisticated weaponry, innate mental structures that allow learning at a rate and with a sophistication that no adult can match. Children do not just memorize facts; they develop, without apparent effort, sophisticated cognitive structures in which to embed knowledge. Nowhere is this more apparent than in a child's acquisition of language. Children learn, subconsciously, sophisticated rules of syntax and the connections between these syntactic rules and underlying semantic structures. A five-year-old child cannot describe the rules for choosing verb tense, creating the syntactic structure of a question, using adjectives and the like, but she can do all of these things and many more. Starting from ground zero, a child learns to understand her native language in less than three years and to produce it in less than five years. This is an astonishing mental feat.

It is by now well understood that young children learn their native language(s) so well because they are equipped with innate mental structures and abilities fitted precisely to this task. Human children have an innate language ability that would be the envy of an adult Einstein or Newton. Why not take advantage of this high-octane ability?

One of the principal theoretical drivers of the Pittsburgh Plan is the desire to enlist this innate machinery in the tasks of learning to read and to do mathematics. We believe that both reading and mathematics have many of the characteristics of an oral language (a notion that is obvious for reading, but perhaps less obvious for mathematics). Thus, the Plan works hard to create an environment in which reading and mathematics can be learned at least in part through the same process by which the child learned oral language.

This idea of treating reading and math acquisition as language acquisition has many consequences for the detailed way in which we structure the homework materials and sessions, all of which are discussed more completely elsewhere in this book. For present "broad-brush" purposes, we want

4 From three words to five.

to focus on one particularly important idea, the fact that language learning results from immersion. A child encounters spoken language every day, for many hours.

We cannot duplicate this type of immersion, but we can understand the importance of regular, repeated exposure over a long time. The homework sessions should not be intense experiences that are often skipped, or that are clustered together and then forgotten for weeks at a time. "Quality" time cannot substitute for "quantity" of time. It is critical that we stick as closely as possible to the schedule of at least four sessions a week, because repetition and frequent exposure are essential to the full and proper functioning of the child's powerful language learning mechanism.

> **" . . . repetition and frequent exposure are essential to the full and proper functioning of the child's powerful language learning mechanism."**

The Pittsburgh Plan is not dramatic or cataclysmic; it is a plan of gradual accretion and imperceptible change. Woody Allen once said, "Ninety percent of life is just showing up," and that is a good summary of the Pittsburgh Plan. Trust the Plan, trust yourself, and most of all trust your child. Just show up; do the sessions four times a week, and do not worry about measuring change. It will occur. If we just make the homework sessions fun and stick to the schedule, the results will come.

Four sessions a week is not a magic number; three or five would probably work just as well. The key is to do enough sessions, regularly, to get the benefits of "immersion," while at the same time not doing so many that the child begins to feel pressure. Four sessions a week seems to strike a nice balance between these two extremes.

Let us incorporate the idea of regular sessions into our definition:

> Third Definition of the Homework Sessions: *Regularly scheduled, relaxed, happy time together.*

We still need to discuss one last question relating to the quality of our homework time: **"How do we help our children develop self-confidence and a love of learning?"**

Certainly, we all marvel at the curiosity and energy of young children. They are fascinated by the world around them; they love to learn. They revel in their mastery of skills, and work very hard at gaining knowledge and understanding. Children love to count, or recite the alphabet, because they can, because it is something that they have mastered that has important applications in their everyday world. A young child's love of learning is so strong that it often survives for a few years, gradually shriveling and wasting, even in the salt fields of elementary school.

Imagine how much richer your child's life will be if she can maintain that energy and joy of learning into her adult years. After all, we are blessed to live in a world that is full of interesting things: the behavior of ants; Bach's minuets; superstring theory; "crossing the chasm;" topspin lobs; trilobite fossils; Picasso; Jupiter; black holes; the histories of Genghis Khan and Caesar Augustus; and much more.

Unfortunately, after being compressed and shaped in generally uninspired classes for twelve years, most high school graduates have lost their curiosity, or at least had it greatly diluted. As a result, they are no longer very good learners, and they have lost one of the great sources of joy in life, something that is a dominant characteristic in most children, namely, the love of learning.

How do we avoid this in the homework sessions? We lead by example! We come to these sessions armed with enthusiasm and excitement, sharing our happiness that our children are learning things that will enrich their lives forever.

What could be more exciting than learning to read? Suddenly, a whole new world opens up before your child, a world populated by Peter Pan and Dorothy, Harry Potter and Sam-I-Am, a world in which the beautiful and the wonderful coexist on a daily basis. And just as importantly, suddenly your child becomes empowered, she has the ability to learn things on her own. Before you know it, a regular part of your daily ritual will involve your child coming to you, book in hand, telling you exciting facts -- tigers can be thirteen feet long and weigh 1000 pounds, the blue whale is larger than the largest dinosaur, people have actually walked on the moon . . . . .

The excitement of reading is fairly obvious. For many people, it is harder to appreciate the joys of understanding mathematics, because so many of us are products of a flawed system that makes math a serious and scary subject. So you may be skeptical about the idea that math can be exciting, at least for a while, but you will soon see that your child loves learning how to add and subtract, how to count backward, and how to connect fractions to shapes. If you can muster up enthusiasm for the math part of the homework sessions, you will find your child to be a willing and vigorous participant.

In short, the Pittsburgh Plan will work much better if you bring your "A" game to the homework sessions – not your "Julius Caesar/I am the big boss" A game, but your "scientist/explorer/artist shared enthusiasm" A game. If you can do this, your child will look forward eagerly to the sessions, and will internalize the idea that learning can be fun.

Taking this into account, we can now state a final broad-brush definition of the homework sessions:

<u>Final Definition of the Homework Sessions:</u> *Regularly scheduled, relaxed, happy, enthusiastic time together.*

So that we do not lose track of them, I would like to restate in summary form our principle objectives for the homework sessions:

Objective #1: Do no harm; keep the sessions happy and relaxed.

Objective #2: Create an environment that encourages the full use of the child's powerful innate language learning abilities.

Objective #3: Stick with it. The Pittsburgh Plan is like a diet; nothing much happens in any one day, but over a period of several years, the results are life-altering.[5]

Objective #4: Support the child's growing and well deserved sense of self confidence.

Objective #5: Help the child retain her love of learning and joy of mastery.

Note that we do <u>not</u> list as an objective "teaching reading and math to the child." This is not an oversight, but instead is a fundamental aspect of the Pittsburgh Plan. We are not teaching; we are instead allowing learning to occur. We keep uppermost in our minds the guiding principle that the homework sessions are not about us, they are about our children.

In a very real sense, we are making a sincere effort to get out of the way. We are creating an environment for learning . . . . but we are not trying to teach. We are providing plenty of emotional support . . . . but we are not dominating the sessions. We are committed and constant . . . . but we are not applying pressure. By following these rules, we avoid the unintentional but very real impediments to learning that accompany most efforts to teach.

If we handle the homework sessions correctly, the learning will take care of itself, and our children will emerge feeling good about themselves and thinking of learning as a fun experience. In this sense, our old-fashioned program gains a New Age credential: our path is indeed our destination!

> **"We are not teaching; we are instead allowing learning to occur."**

5 Please note two crucial difference between the Pittsburgh Plan and a diet, however: (i) the Pittsburgh Plan is fun; and (ii) under the Pittsburgh Plan, you only have to be good for fifteen minutes a day. If we could develop a diet that had the latter characteristic, we would go down in history.

Chapter 8: *Pint-Sized Einsteins*

Because my wife is a professor, I have been around university faculty for almost three decades. In that time, I have come to know many brilliant academicians who have moved to the United States from non-English speaking countries such as India, Germany, China, Japan, Brazil, Vietnam, Spain, Turkey, and Scotland; in fact, it has been my good fortune to know geniuses from almost every academic discipline and from every corner of the globe. In most cases, these brilliant adults came permanently to the United States as graduate students or as very young post-graduate students; i.e., in their early to mid-twenties.

There is one interesting common trait shared by all of these highly intelligent people: significant difficulty in speaking English. Probably you have noticed this as well. It is very hard for an adult to become proficient at speaking another language. It seems to me, based upon my own experience and investigation, that this type of adult language-learning difficulty extends as well to adult learning of mathematics and computer skills, at least for people who did not gain basic competency in those areas at a young age.

Now, compare the language-learning performance of a young child. Starting from absolute ground zero, the average American child – in fact, every child that is not learning disabled or enormously deprived in some way -- learns to understand English by age two or three and to speak it fluently by age six, usually much earlier. In other words, children who will grow up to be completely average intellectually routinely outperform adults such as Albert Einstein, Henry Kissinger, and even Jackie Chan in the tremendously important intellectual task of learning to speak English.

And that is not all. A child who grows up in a multilingual environment will easily learn two, three or even more languages, and will speak them all as a native speaker. Try this for yourself; try to learn, say, Japanese and Arabic at the same time, and see how well it goes!

Of course, the idea that children learn more easily than adults is not exactly news. We all know that you cannot teach an old dog new tricks. But most of us forget just how astonishing and powerful a child's language learning ability can be. And we also seem to forget that the child does her learning mostly on her own, outside of any classroom or formal instruction, by being immersed in language and absorbing its rules from exposure to them. Adults help children learn language, but do not explicitly teach deep technical rules for processes such as forming past tenses of verbs or creating conditional sentences ("If you cook spinach, I will not eat it."). Instead, children learn these rules on their own, inductively -- that is, by extracting the rule from their experiences of hearing such words and sentences. In fact, language rules seem to be learned subconsciously. For example, most of us, children and adults alike, have some difficulty articulating the rules governing the use of related verb structures such as "I went," "I was going," "I have gone," etc., but we have little trouble using them correctly.

Often, when I talk about the great ability of children to learn language, people respond by saying, "So what? Of course they learn more easily; they are writing on a blank slate." The first response to this argument is that it is at least incomplete, and is probably wrong. But a second and more important response is that whether or not this argument is true, it misses the point. This is not some sort of a contest between children and adults where we want to establish a fair handicap; this is an effort to harness the undoubted power of a child's mind to help her learn.

Fair enough, you say, but how do we do that? What can we extract from our observation of a child's language-learning behavior that will help us teach reading and math, the two main academic targets of the Pittsburgh Plan?

First, a very important principle: *If a child actively extracts a rule, whether it be a rule of language, reading or math, she owns it, forever and deeply; if she passively absorbs a rule, her knowledge is shallow and precarious.*

Why is this true? What is the difference between struggling to figure something out and having it handed to you on a silver platter? Is it not more efficient to save the child the effort of extracting knowledge herself, and instead give her the benefit of the hard work that has been done by others? After all, this is how human culture has advanced over the millennia, each new generation benefiting from the knowledge and discoveries of the ones that went before.

The answer lies in the fact that intelligence is not so much what we know as it is how we know it. When we help a young child learn we are not just loading data into the memory of a computer; we are actually building the computer. Not only are we helping the child learn facts; we are also helping her learn to organize those facts effectively so that they can be retrieved and used when needed. And the organization of knowledge has an even more important consequence; it determines how well the child will be able to see connections and understand relationships between various facts. So when a young child learns a fact, she is doing more than just burning it into her memory; she is establishing a network of connections between that fact and her other knowledge, a network that will determine how well she can use her knowledge. Intelligence is much more the ability to draw connections and understand relationships than it is the mere memorization of facts.

"Intelligence is much more the ability to draw connections and understand relationships than it is the mere memorization of facts."

What does this mean for our purposes? When a child struggles to extract a rule or connection, she integrates it into her mental framework. In so doing, she alters and strengthens that framework, a good thing (she now has a stronger CPU in her computer). And, the new rule will be connected to her existing knowledge in a sensible way, because it was necessary for her to establish those connections in order to extract the rule. By making her work, we help her strengthen her internal architecture, and we also ensure that her new knowledge is well-integrated with the rest of her knowledge. On the other hand, if the child is merely a passive recipient of information, she simply stores it in her memory without an elaborate network of connections. Not only is such knowledge less easily accessible at later times, it is also less well understood, because the richness that comes from an understanding of relationship and context is missing.

Although we do not understand cognition very well, we have learned enough to get fascinating glimpses into the workings of memory and thought. A fairly well-known theory states that knowledge is stored in multiple locations in the brain, with more important facts being stored in more locations. This idea, one version of something known as a "neural network," is interesting because it suggests that storage of facts is not nearly as important as the ability to draw connections and retrieve facts – the brain can afford to "waste" multiple storage sites in order to facilitate connections to the knowledge in question.

One of my friends has implemented this idea of neural networking in his own home library. It is perfect for him because he loves to read, is highly disorganized, and is quite wealthy. Just as the human brain can be profligate in "spending" memory sites, Erwin[1] can be profligate in buying books and building bookcases. Erwin's procedure is simple. When he wants a book that he thinks he owns, he looks through his library for about 45 seconds. If he cannot find it within that time period, he calls the bookstore and orders another copy, which he then shelves randomly. As a result, Erwin's library has multiple copies of many books – the more he likes a book, the more copies he owns!

When a young child learns language, she learns it from experience, from repeated exposure to information. These multiple exposures allow her to notice repeated patterns and generalize them. A child learns the meaning of most words through this process of generalization from repeated exposure – a process that is known more formally as "induction." For example, a child may hear adults talk about "chairs" on multiple occasions, each time noticing an emphasis upon an object with four legs and a seat, an object upon which people sit. But perhaps an occasional note of confusion will be thrown into the mix, perhaps during a hike through the woods a parent will spot a fallen log and exclaim, "There is a comfortable chair!" and sit down on it. Or the family will buy a bean-bag chair! The child will ultimately develop a very rich understanding of the word "chair" – perhaps a slightly fuzzy definition, satisfied most clearly by the traditional kitchen chair, a little less comfortably by the overstuffed living room armchair, and even less comfortably by the beanbag chair. And, perhaps the child will come to understand that "chair" can even be used, in appropriate contexts, to mean "anything a person can sit on," such as a log in the forest. Thus, for the "inductive" child, the word "chair"

1 Do I really need to note that "Erwin" is not his real name?

becomes a powerful linguistic tool, full of meaning, with a richness and depth of content that goes far beyond a 'picture-book" definition that a child might learn in a classroom. It is this far richer idea of chair that allows the word to expand and find new meanings in new contexts:

- "I will *chair* this meeting;"
- "Professor Dweeble holds the endowed Bozo *Chair* for Educational Psychology at the University;" and
- "My favorite pop group was Sonny and *Chair*."

OK, I am just kidding about the last one, but you get the idea. Language derives much of its power from its richness and flexibility. Even a commonplace definite noun like "chair" has hidden depths. And this power, this flexibility, is intimately connected with the process by which we learn language. In fact, it is not a giant leap to suggest that our innate, inductive ability to learn language bestows this power upon the languages that we learn; the richness comes about largely due to the fuzziness of inductively learned knowledge and our apparently innate ability (at least at a young age) to impose a meaningful structure – a network of connections and relationships – upon this inductively learned material.[2]

"Language derives much of its power from its richness and flexibility."

Our innate language learning apparatus is not just an ability to memorize facts, but much more. It is an ability to organize material into a framework that enables the creation of a vast web of connections between words. In a sense, this apparatus seems to impose a structure (a "syntax") upon the underlying semantics of a language – and, perhaps even more powerfully, to decode and internalize a syntax that already exists within the language. Stated more elegantly, a young language learner seems to be hard-wired to decode the syntactic structure of a language, and to learn and incorporate that underlying structure into his own mental framework. It is interesting and important to note that this innate, hard-wired ability seems to disappear gradually beginning at about the time of puberty.

A key idea of the Pittsburgh Plan is the notion that this ability can be used to learn math at a young age. After all, math tracks language in many important respects. It has underlying content (semantics) that is mapped onto a syntactic structure, which itself provides a great deal of added meaning by providing a network of connections and relationships. And, like language, it has a lexicon – the basic mathematical facts of the discipline.

2 Ludwig Wittgenstein revolutionized philosophy by pointing out that many "philosophical questions" are actually meaningless, merely going to the definitions of words – E.g., what is "truth?" -- that are learned inductively (and thus differently by different people). To the extent that a philosophical issue boils down to a disagreement over the definition of a word, it is not a very interesting issue! Wittgenstein, Philosophical Investigations (1953).

Let me make this a bit more clear. Syntax can be thought of as the set of rules that gives meaning to the order, or sequence, or structural relationship between the words of a language or the symbols of a sentence or equation. Thus, in English, the basic declarative sentence structure is SVO – Subject, Verb, Object, such as "John hit Sally." This basic syntactic rule allows a speaker or reader to understand sentences very rapidly, without parsing the words carefully to ascertain which is filling which role. Thus we immediately know the difference in meaning between "John hit Sally" and "Sally hit John," even though all of the words are the same, only the order is different. [3]

These types of syntactic rules are also very important in all branches of mathematics, even in basic arithmetic. As with a sentence, much of the meaning of a mathematical expression is syntactic: "8/4" means "8 divided by 4;" "4/8" means something entirely different. The importance of syntax in mathematics can be seen more generally from the oft-repeated observation that the essence of mathematics is notation – i.e., the technique by which syntactic structure is displayed. In fact, this is true of physics as well; the famous Feynman diagrams that did so much to advance quantum field theory in the mid-twentieth century are nothing more than a simplified notation for some very complicated mathematical expressions. [4]

Why not free our children to use their inductive powers – their innate syntax-decoding abilities -- on things other than oral language? Why not enlist their language learning apparatus in a broader range of situations, such as the learning of mathematics?[5] Releasing the shackles can lead to some amazing results! For example:

- An eight-year-old, presented with a series of simple word questions (e.g., "How many different ways can you line up five different color crayons?"), derived the general formula for the number of permutations of n objects and also came up with the idea of a factorial. [6]

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3 Of course, language is complex, and different syntactic rules come into play when more complicated constructions are used – e.g., "John was hit by Sally."

4 Feynman enjoyed driving all over the American west in his van, which was painted with many Feynman diagrams. In an odd and possibly apocryphal encounter, Feynman stopped for gasoline in a tiny Nevada town (population six), where the gas station attendant just happened to be a graduate student of theoretical physics who was taking a sabbatical from his intense studies to regain his equilibrium. The attendant shocked Feynman by recognizing the diagrams, saying, "Those are Feynman diagrams!" Feynman then shocked the attendant in turn by replying, "Yeah; I'm Feynman." They looked at each other for a Zen moment or two, then Feynman drove off.

5 The idea of "language" can similarly be extended to include music, which is also a means of communication having a syntax – i.e., a set of rules governing allowable or proper combinations of notes. Recent research suggests that (i) the same region in the frontal lobe enables proper construction of the syntax of both language and music, and (ii) musical training, especially at a very young age, is linked to significant increases in the volume of the auditory cortex and the strength of the response of the relevant regions of the brain to musical tones. In other words, early training in the "language" of music actually seems to produce significant physical changes in the brain, changes which help the brain process musical tasks more efficiently. Norman M. Weinberger, "Music and the Brain," Scientific American, November, 2004. Can such findings be extended to early training in mathematics?

6 Do not worry; you do not need to understand these concepts to deploy the Pittsburgh Plan! For those who are interested, the number of ways that n different objects can be put in order is equal to n!, which stands for "n factorial." The meaning of n! can best be taught inductively: $2! = 2•1$; $3! = 3•2•1$; $4! = 4•3•2•1$; $7! = 7•6•5•4•3•2•1$, etc. See how well induction works? The standard, non-inductive way to teach n! is to say $n! = n(n-1)(n-2) \ldots •2•1$

- A five-year-old beginning kindergarten student came up on her own with the distributive law as a way to solve a subtraction problem (she realized that "20 – 15" can be thought of as "4 fives - 3 fives" which equals "1 five").
- A six-year-old was helping to give cups of water to the runners at the halfway point of a 10K race. As the leaders came by, she ran out and offered each one a cup of water. Because it was a very hot day, the first few runners grabbed the cups and splashed the water onto their heads. As the next runner approached, she ran out and, to the runner's astonishment, dashed the cup of water into his face! So we see that induction has its dangers as well as its benefits.

A child who has learned inductively will tend to have multiple perspectives, i.e., different angles and views of the same underlying concept. This will allow her to draw connections and see relationships that a more didactic learner might miss. Also, a child who learns inductively is free to tackle material using her own strengths. One child might think spatially about a math problem, while another might think logically, in a series of steps. One young reader might prefer a phonics approach to an unfamiliar phrase while another might lean more heavily on context. Each child is free to fit new information into her already-well-developed mental framework, instead of being locked into a rigid approach and a predetermined set of connections that is imposed on her – a set of connections that may well be much harder for her to internalize, since the didactic way in which they are presented may not enlist the aid of her innate language learning apparatus.

> **" . . . the Pittsburgh Plan does not prescribe very much at all about the way that a child should learn. This is mostly left up to the child!"**

By now you see that the Pittsburgh Plan does not prescribe very much at all about the way that a child should learn. This is mostly left up to the child! The parents and the Plan materials provide a good deal of behind-the-scenes guidance, and erect a carefully structured learning environment that is designed to make maximum use of the child's abilities. But we are agnostic when it comes to theories as to how the human mind learns math. We do not really care which method a child uses; we just want to make sure that we do two important things: (i) provide a rich variety of materials that engages the child's full learning apparatus and ability; and (ii) get out of the way so that we do not block creative learning by our preconceptions.

So how do we do this? **How do we enlist the child's learning mechanisms for our math-learning purposes?** There are six main principles that the Pittsburgh Plan has developed to accomplish this goal. These principles will be fleshed out in detail in Part 3 below, but are briefly summarized as follows:

1. Provide Rich and Varied Content. Your child cannot establish connections unless she has things to connect. Ensuring the mathematical richness of the content requires a bit of math expertise, but you need not worry about this; we do this for you by structuring the homework materials so that a wide variety of important mathematical relationships are embedded within them.

2. Give Guideposts Rather Than Instructions. The idea here is to avoid just telling a child important math rules; instead, we try to give the child hints, or guideposts, so that she can discover the rules for herself and thus make them her own. We do this very patiently, in layers. (I should emphasize that I am explaining here for the interested reader one part of the theory governing how the Homework materials are structured; none of this kind of analysis appears overtly in the materials, and in particular parents using the Plan are not expected to understand this theory, or even care about it! Remember that Parts 1 and 2 of this book are entirely optional. You can do the Plan quite nicely without reading them.)

Let me illustrate this layered approach with an example, using outline format to show the layers.

A. Highest Level Skill – Multiple Perspectives. One of the key highest-level math objectives of the Plan is the ability to approach problems from multiple perspectives. We do not teach this by saying to the child, "It is important that you learn to approach problems from many different perspectives." That would be didactic and largely wasted, even on an older child. Instead, we approach this objective from the bottom up by helping the child learn to think about ideas in multiple ways in many different specific contexts.

1. Level 2 Skill -- Multiple Perspectives in the Division Cluster. One (of many) such specific contexts is what we call the "division cluster" of ideas – the ideas of fractions, division, decimals, ratios and percentages, each of which is really just a different way of thinking about the same underlying content. Again, we do not begin by telling the child, "Fractions, division, decimals, ratios and percentages are all really just different ways of thinking about the same thing." Instead, one by one, we allow the child to make the connections between these ideas, as shown below with respect to fractions and division. Then we use series of problems and open-ended discussion questions to encourage generalization to the division cluster as a whole.

(a) Level 3 Skill -- Fractions and Division Are Two Different Views of the Same Thing. For example, we help the child discover that fractions and division are really just two different ways of thinking about the same thing, two different names for the same idea, much as "Mary Samantha" and Sammy" are two different names for the same child. By now, you probably can guess that we do not do this by saying, "Fractions and division are really just two different views of the same thing." Instead, we give the child a series of problem pairs: simplifying 4/2 and doing 4÷2; again for 6/3 and 6÷3; again for 10/2 and 10÷2; and so on. If necessary, we ask leading questions: "Do you notice anything interesting here? Do you see any connections between these division problems and these fraction problems?" Similarly, we help the child discover that fractions and decimals are two different views of the same thing, and so on for each pair of elements of the division cluster.

In short, although our dedicated team of course developers (a euphemism if there ever was one!) created the homework materials from a top-down perspective, based on a fairly sophisticated understanding of mathematics, the Plan "teaches" from a bottom-up perspective, beginning by allowing the child to discover a number of Level 3 skills and eventually helping her to generalize her way up to the higher-level ideas. And, most importantly, at each level we avoid "teaching" the key ideas and instead do our best to help the child discover them – i.e., we provide hints, or guideposts.

Let us emphasize again that this theory is already embedded in the Homework materials – there is no need for the parents to understand it or worry about it unless they find it interesting (although one of our hopes is that expert parents will help improve the Plan as they implement it).

3. Provide Repetitive Content. Every parent knows, usually from hard time served, that children love repetition.[7] There is a reason for this! As we have discussed to the point of beating it into the ground (because repetition is also good for adults), children learn best by induction, and in order to do the hard work of induction, they need repeated and lengthy exposure to information. It is easy to remember something; it takes a while to understand it and integrate it into a network of connections. In a sense, a child's need for repetition is strong evidence for the importance of inductive learning.

4. Encourage Creativity. Creativity is the A-Number-One, Grand-Prize-Winning, Blue-Ribbon, Best-In-Show, most important cognitive skill. It is the Holy Grail of education. So we must remember to honor it and encourage it when we encounter it in our children. For example, if a child comes up with an approach to a problem that is correct but cumbersome, or that is intelligent but fails for some technical reason, we must be very careful not to squelch such efforts by dismissing the approach or by trampling on it with a rapid presentation of the correct or best approach. Instead, we praise the effort and, if necessary, break the news of inaccuracy gently.

5. Encourage Multiple Perspectives. We discussed this above, but is worth emphasizing that multiple perspectives is not only a high-level skill objective of the Plan; it is also an important teaching technique.

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7 It is one of my fondest dreams that I will never, ever have to read Pat the Bunny again!

6. Build a Framework Rapidly, Then Hang Facts On It. Math teaching in elementary school is largely the teaching of facts – number recognition, addition facts, subtraction facts, etc. The problem with this approach is that it does not give the child many opportunities to draw connections. The child needs to have an arsenal of organizing principles, an internal framework, upon which to hang these facts.

The Plan works very hard to introduce operations, principles, and relationships as early as possible in order to facilitate this kind of deep-level, inductive learning of facts. For example, we teach addition as soon as the child can recognize a few numbers.

The philosophy of the Plan is simple: when you have an Einstein on your team, use her! We structure the learning environment of our homework sessions so that your child has the laboring oar because we trust her abilities; this is probably the biggest single distinguishing characteristic of the Plan.

PART 3: NUTS AND BOLTS OF THE PITTSBURGH PLAN

Chapters 9 – 11 describe in detail the first three Phases of the Pittsburgh Plan. We have tried to leaven the procedural detail with occasional anecdotes, but it must be admitted that at their core these three Chapters are a user's manual -- a blow-by-blow discussion of the essential first steps on the road to math and reading success for your child. Be of stout heart, and persevere! It will be worth it.

CHAPTER 9:
Getting Started

Congratulations! If you are reading this you have decided to give the Pittsburgh Plan a try. You will never make a better decision.

How do we get started?

The basic activity under the Plan will be "homework sessions" -- brief periods of time spent seated at a table with your child, usually with your child on your lap. You may recall our final definition of a homework session from Chapter 7:

Regularly scheduled, relaxed, happy, enthusiastic time together.

This is a good general guideline, but now it is time to get more specific. In fact, it is time to begin the Pittsburgh Plan!

The Place. The first thing you need to do is identify a place for the sessions. The beginning parent always has the same notion: a quite, studious place, far from the noise and confusion of regular family life; a place where the serious business of learning can take place unencumbered by distractions and interruptions.

If this is your initial thought, it shows that you are taking this project seriously, which is good . . . . but you have to lighten up a bit! This is not serious business, it is fun! And it is not a classroom experience, it is learning by immersion, like learning a native language.

Do the homework sessions in the kitchen, preferably at a time when the whole family is around! If Dad is handling a homework session, Mom should be sure to notice a correct answer and applaud it, and perhaps jump in and join the fun for a minute or two. Brothers and sisters are welcome to watch, and even sit on the same lap during the session. They should not horn in too much, because the sessions are designated fun times for the student, but only good things come from having siblings interested and peripherally involved. (For example, a sibling's interest enhances the perceived value of the session for the child in the spotlight. Also, younger siblings will be eager to have the chance to start the program themselves when they are old enough.)

The Time. Next, you need to pick a time for the sessions. We suggest that the sessions take place in the morning, after breakfast but before starting the days' activities. There are many reasons for this preference.

- The homework sessions are important, much more important than most of the day's activities. Thus, you want to make sure that they do not get squeezed out by the other events of the day. Doing them first thing in the morning helps ensure that they are not lost in the shuffle.

• Ideally, the time should not vary too much from session to session – the sessions should be a part of the child's regular routine, four times a week. In our experience, it is easier to build a stable schedule in the morning than at any other time during the day.

• As mentioned above, the sessions should take place in the middle of the active life of the family, and for many families morning is the time when family members are most likely to be at home together.

• Finally, it is important that both you and your child be fresh and enthusiastic for the sessions, which pretty much rules out the evening for any parent of a small child!

Often, parents are able to create the fifteen morning minutes needed for the homework sessions by simply driving the child to school or preschool instead of having her take the bus. Of course, this creates time for the sessions in the child's schedule, but the quid pro quo is that it puts more of a time burden on the parent. The Pittsburgh Plan tries to be up-front and honest about this: it is an unavoidable fact that following the Plan will require parents to allocate time to the Plan that otherwise could be used for something else. TANSTAAFL -- There Ain't No Such Thing as a Free Lunch – if you want your child to excel at school and develop a lifelong love of learning, you have to sacrifice an hour or two a week!

"TANSTAAFL -- There Ain't No Such Thing as a Free Lunch – if you want your child to excel at school and develop a lifelong love of learning, you have to sacrifice an hour or two a week!"

Although the morning has always seemed to us to be the optimal time for the homework sessions, you should feel free to pick any time that works for you and your child. The key, though, is consistency: four times a week, at a standard designated time. Our experience suggests that if you do not designate a consistent time and stick to it, the sessions will eventually fall by the wayside. There will always be something more immediate and pressing – the big sales report due the next morning; practice for the piano recital in three days; the new special program about dinosaurs on PBS; etc.

The great power of the Pittsburgh Plan is that it is accretive in nature, cobbling together significant cognitive gains from a large number of small steps. This is also its great potential flaw, because we live in a fast-moving world where results are tallied on a daily basis and long-term projects get lost in the press of more immediate events. In order to succeed at the Plan, parents must learn to make time their ally rather than their enemy. In other words, they must be able to see the value of a long-term, incremental program, and stick with it in the face of alternative pressures that seem more immediate.

A Comment About Tone. Children are very smart – that is one of the fundamental tenets of the Plan -- and they are sensitive to subtle signals sent by their parents. This creates both an opportunity and a risk. The opportunity? If you show by your demeanor and actions that the homework sessions are important, your child will treat them with respect and ultimately will benefit more from them. The risk? If you merely pay polite attention to the homework sessions, and give greater emotional emphasis to other activities such as sports, your child will understand that those other activities are more important to you than the homework sessions, an understanding that will be reflected in the child's benefits from the Plan.

For example, if you spend great amounts of time enthusiastically participating with your child in sports activities, and praise her repeatedly for them, or emphasize them in conversations with others, your child will come to understand that you care deeply about her sports performance. If at the same time you rarely discuss the day's homework session with others, and do not join in the sessions from time to time, then your child will understand that the homework sessions are less important to you than sports; she will participate with less vigor in the sessions and ultimately derive less benefit from them. And, if we want to be honest with ourselves, we will see that the child is right! Whatever we may say, and even think at some surface level, if we are behaving that way, it is because at a visceral level we do in fact care more about our children's successes at sports than at the homework sessions - a fact that is as hard to deny as it is to explain.

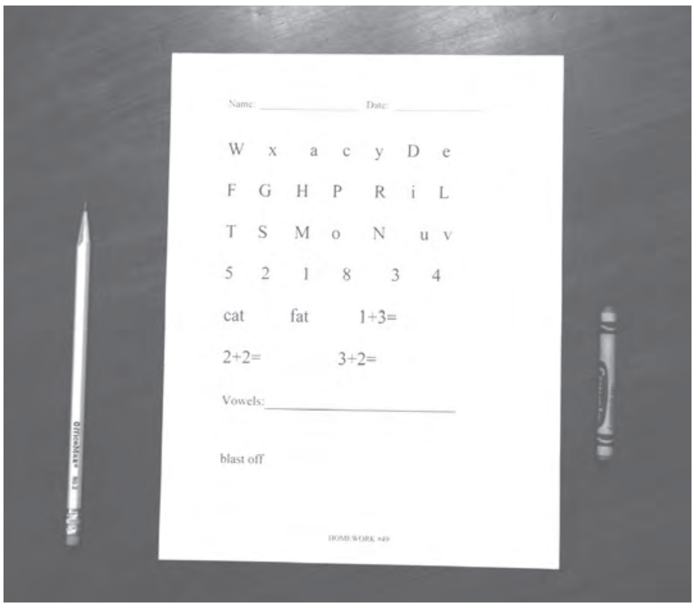

FIGURE 9.2: A complete inventory of all the equipment required to use the Plan with your child.

So, treat the homework sessions as an important part of your family life. Talk about them at the dinner table, report on them to grandparents, praise your child for her work in them, show enthusiasm and a desire to take part in them, in short, treat them in the same way that you would treat her performance as a star player on a soccer team.

<u>The Equipment.</u> What equipment is needed for a homework session? Three things: a homework worksheet; a pencil for you; and a pencil or crayon for your child (let her pick, subject to safety concerns). See Figure 9.2. You can get started by using the homework materials in Appendix A ("Sample Homeworks"), or you write your own homework sheets, using the Sample Homeworks as a guide.

Your Child. Your child should generally be at least twenty-two months old to begin the sessions, although if you keep things fun and avoid placing pressure on her, there is no harm at all in starting too early. We discuss below how to handle the early homework sessions to determine if your child is ready to begin the Plan on a four-day-a-week basis, or instead needs a more gradual or delayed introduction to the Plan. For example, although the student (Sam) whose homeworks are provided as the Sample Homeworks in Appendix A began the program at age twenty-two months, her homework sessions were intermittent and not regularly scheduled until she reached age twenty-eight months.

At its most basic level, the Plan consists of only two elements. First, you and your child should have several fifteen-minute sessions each week in which you work together on the Sample Homeworks, or similar homework sheets that you prepare, in any way that you choose. Second, you should make sure that these session are low-key, low-pressure, and fun.

Although there is a great deal of effort and analysis underlying the Plan, much of that work has been directed at making the Sample Homeworks relatively self-sufficient. You do not need to understand math or the theory of the Plan in order to use the Plan. You do not need to read this Chapter, or the rest of the book. You will help your child a great deal if you simply have four low-pressure homework sessions a week, talking with her about the Sample Homeworks.

That said, it will probably help you do an even better job of implementing the Plan if you understand the underlying structure of the Sample Homeworks. Part 1 of this Chapter describes that structure.

Also, it is possible that you will derive some benefit from looking at the helpful hints that we have developed over sixteen years of experience with homework sessions. To that end, Part 2 of this Chapter consists of a detailed, step-by-step discussion of the sessions involving the first set of Sample Homeworks.

And, of course, you should feel free to use the guidelines and instructions contained in the Quickstart Module – Appendix B – to help you get started with the first ten Sample Homeworks!

Part 3 of this Chapter discusses the second set of Sample Homeworks, called the Early Consolidation Phase. This critical Phase lays the initial groundwork for reading and for the first mathematical operations, addition and subtraction.

The fourth and final part of this Chapter discusses the very important idea of Field Work.

Again, it is worth emphasizing that this Chapter, and in fact most of this book, is not a prerequisite to doing the Plan with your child. It is merely intended to provide additional information that may increase the benefits your child derives from the Plan.

PART 1: The Underlying Structure of the Sample Homeworks

The Big Picture. The homework sessions are divided into Phases. To be honest, the dividing lines between the Phases are somewhat arbitrary because our basic approach is to jump around a great deal, often introducing a new concept only to abandon it for a while, or reviewing old concepts from earlier Phases. Nevertheless, the use of Phases as an organizational principle (one that is, by the way, invisible to the child) is worthwhile because it helps ensure that in the end we are providing materials that cover well-organized and clearly articulated objectives.

We will begin, appropriately enough, with the first Phase, and will proceed step-by-step until we have completed the last Phase. This process will probably take somewhere between three to four years, depending in large part upon the entry age of your child, and will in the end produce as its output (we hope!) a child who: (i) reads very well, far beyond her years; (ii) loves to read (a different thing than reading well); (iii) enjoys math; (iv) is comfortable tackling unfamiliar math concepts or problems; and (v) is substantially ahead of grade level in math and reading.

If we can achieve all of this, I think you will agree that we have done great things. Your child will be on track to be a dedicated life-long learner and a capable academic performer.

Perhaps you are wondering what happens next after your child has completed the Plan? It is a bit premature to tackle this question now, when we have not even started the first homework session! For present purposes, it is probably sufficient to note that there are three main pathways available to a graduate of the Pittsburgh Plan (remember, the graduate might well be only six years old):

(i) moving into the mainstream school curriculum, possibly with some advancement (especially in math), and enjoying the cognitive and emotional benefits of great competence in comparison with peers and the expected curriculum;

(ii) the same as (i), but with added "sideways" or "lateral" enrichment in math; and

(iii) the same as (i) but with continued rapid progress in math using the Pittsburgh Plan techniques.

The question of appropriate next steps following completion of the last Phase is addressed in more detail in Chapter 14.

Overview of the Phases. In general, the homework materials for each Phase follow a structured plan in an unstructured way. Put another way, each Phase has definite objectives that are carefully spelled out and targeted, but these objectives are typically approached multiple times in multiple ways, with significant review and jumping forward and backward. This jumping around frequently occurs across different Phases, to the consternation of our more organized parents but to the general delight and benefit of our children.

Think of it as if we are setting up a playground. We plan the selection of the equipment – swing sets, teeter-totters, sandboxes, etc. – carefully, paying attention to the physical capabilities of the children and also to the skills we wish to help them develop. And, of course, overriding everything else, we ensure that each piece of equipment is safe. Then, after putting a big fence around the playground, we cry havoc and let slip the dogs of play! In less Shakespearean terms, we turn the children loose to play on the playground as they wish. And, apart from occasional coaching on techniques

" . . . we cry havoc and let slip the dogs of play!"

of swinging, digging, etc., we stand off to the side where we can monitor things without interfering too much with the activities.

This picture pretty well describes our approach to each Phase. For each Phase, we will provide you with the following:

(i) a specific set of objectives (the fence around the playground);

(ii) a complete set of relevant homeworks from the Sample Homeworks in Appendix A (playground equipment); and

(iii) a set of detailed suggestions and comments tied to the Sample Homeworks for that Phase (comparable to coaching a child on how to "pump" on a swing).

And, we will ask you to keep the homework sessions low-key and relaxed – the equivalent, in our playground analogy, of making sure that the playground equipment is safe.

Although we recommend that you follow the Sample Homeworks and detailed suggestions fairly closely at first while you are developing a feel for the dynamics of the homework sessions, you are completely free at any time to vary the program within the defined objectives. In fact, it is your session, and your child; you are free to vary the sessions in any way you think appropriate! For example, you may opt to do review sessions with prior homeworks, or to give the same homeworks several times in a row. You may decide to skip homeworks as being redundant or boring for your child, or to make up some of your own homeworks, guided by the Sample Homeworks and perhaps emphasizing items that are giving your child a little trouble. There is no fixed time frame for a Phase, and there is no rule against reverting to materials from a prior Phase.

"Let me say that again: repetition is a powerful learning tool."

This is an important enough point to reiterate in different words. The Phases are defined by their objectives, not by a specific set of Sample Homeworks; they can consist of a few homework sessions or fifty or more – it all depends upon you and your child. Although we will discuss the Phases by referring to the Sample Homeworks in Appendix A, you should feel free to do more or less homework sessions in each phase, depending on your assessment of your child's progress and needs. One common trick, which works very well, is to use the same homework sheets multiple times, since repetition is a powerful learning tool. Let me say that again: repetition is a powerful learning tool. Also, it is quite simple to modify Sample Homeworks to create new homeworks aimed at the same objectives.

As noted above, the Sample Homeworks are by no means linear in their approach to introducing new ideas. Instead, there is a great deal of hopping around and introduction of new concepts followed by their temporary abandonment. This tracks the basic theory underlying the Plan, namely, that although we do not fully understand how learning takes place, we do know that children are very good at it, especially when it comes to learning a native language, so we do our best to provide a rich and varied learning environment similar to that which is present when a native language is learned. And then, we do our best to get out of the way! We do not let our assumptions or preconceptions limit the child.

When we discuss the Phase 1 objectives in the next section of this Chapter, you will see that we take this idea of a rich learning environment very seriously. The foundation of Phase 1 (as with all of the Phases) is a cluster of concrete learning objectives of the usual "pre-school" type – things such as learning to recognize letters and shapes. But lying beneath these concrete objectives is an extra layer of material, a subterranean current of important abstract concepts that find concrete expression in our mundane homework tasks. We do not present these abstract concepts directly to the child; certainly, we do not expect the child to understand them at this point! But we do expect that repeated exposure to concrete expressions of these concepts over a long period of time, with a bit of gentle discussion and guidance, will help the child eventually come to internalize these important concepts and "own" them as a powerful part of her intellectual arsenal.[1]

Note that you do not have to understand these abstract ideas yourself in order to allow your child to learn them in this fashion! That is the beauty of the Plan's approach: by relying on the child instead of the parent or teacher, we can break the vicious cycle of math-anxious parents and teachers begetting math-anxious children. It will suffice if you simply work through the Sample Homeworks with your child and guide the discussions in those sessions as described in this book.

Keeping in mind this fundamental approach -- relatively unstructured homework sessions aimed at well-defined objectives – we are now ready to describe the first Phase, known colloquially as the "Toe-In-the-Water Phase." We will walk you through this Phase in excruciating detail, because it is your first exposure to the way the ideas of the Pittsburgh Plan are implemented.

PART 2: Detailed Discussion of Phase 1: The Toe-In-the-Water Phase

Sample Homeworks: Sample Homeworks #1–14 for a sample child named "Sammy."

Objectives: The Sample Homeworks for Phase 1 are simple pages containing only three things: a line for the child's name and date; a few letters; and a few simple shapes. There are actually more objectives for this Phase than there are homework tasks!

A few of the objectives contain technical ideas. Remember, you do not need to know or understand – or even read! -- the objectives in order to teach your child under the Plan. Why? Because we are not really asking you to do anything very complicated – just help your child work through these simple homeworks, and allow her to learn. It should relieve you to learn that we are relying on your child, not on you!

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1 As the great John von Neumann said, "In mathematics you don't understand things. You just get used to them."

In any event, here are the objectives for the Toe-In-the-Water Phase (Phase 1):

- Help the child become comfortable with the idea of a homework session;
- Assess whether the child is ready for regular sessions or instead needs to go through a period dominated by Field Work (see Part 4 of this Chapter, below) with only occasional homework sessions, perhaps once every week or so;
- Learn seven simple capital letters -- in the particular case of the Sample Homeworks, B, O, M, S, A, T, and E;
- Lay early groundwork for reading;
- Learn four basic shapes: circles; squares; rectangles; and triangles;
- Basic counting with one-to-one correspondence to twelve;
- Recognize the child's printed name;
- Introduce the idea of a calendar date;
- Gradually increase the number of homework tasks (e.g., from two rows of three letters to five rows of four letters);
- Introduce the key mathematical ideas of "inside," "included in," and "bounded;"
- Introduce concrete examples of the important abstract idea of symmetry;
- Lay the groundwork for the idea of defining mathematical objects by lists of characteristics (a triangle has three sides, etc.);
- Introduce the idea that things (circles, squares) can be thought of in more than one way;
- Introduce the idea of nested or hierarchical sets (squares are a type of rectangle); and
- Help the child become comfortable using a pencil, pen or crayon on the homework sheet as part of the sessions.

Time Frame For Phase 1. Sammy began Phase 1 at age 22 months and finished seven months later, at age 29 months. Each Sample Homework was used, on average, twice during this period, meaning that homework sessions occurred roughly once per week. During this period, Field Work was done extensively, until Sammy finally seemed ready to tackle homework sessions more frequently.

How can you tell if your child is ready for regular homework sessions? This is a judgment call on your part. If your child is having fun, it probably does not matter whether she appears to be learning anything – it would make sense to do the homeworks regularly, several times a week. The apparent lack of learning might be deceiving; a lot of the action goes on well below the surface. On the

"The apparent lack of learning might be deceiving; a lot of the action goes on well below the surface."

other hand, if your child seems completely uninterested in the homework sheets, and just struggles to get out of your lap, then probably you should focus on Field Work and merely try one homework session a week as a check to see if she has become more interested in the sessions.

***Detailed Suggestions and Comments for Phase 1*:** As you go through the following detailed suggestions concerning the Phase 1 Sample Homeworks, it would probably be a good idea to have a copy of the relevant Sample Homework sitting in front of you for easy reference. Please note that at the end of the list of detailed suggestions, there is a transcript of an illustrative session using Sample Homework #1.

We begin with two general suggestions and then move to a more specific discussion of Sample Homework #1:

1. ***Field Work.*** Young children are oriented toward tactile sensations – touching, feeling, and even tasting. Part of the job of the early Phases is to build a bridge from your child's early, mostly tactile experience with written language to written language in its more conventional form, i.e., as letters and words written on paper. In order to build this bridge well, we need a strong foundation – that is, we need your child to have had a rich exposure to letters, numbers and counting in tactile forms such as letter blocks, plastic letters, flash cards, letters on signs, etc.

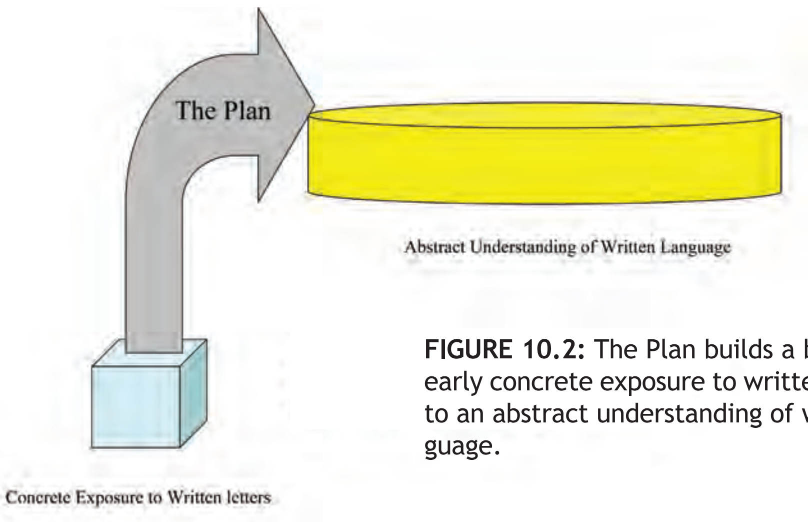

FIGURE 10.2: The Plan builds a bridge from early concrete exposure to written language to an abstract understanding of written language.

It would be best if your child came to the initial homework sessions already having experienced a significant amount of this type of "touching and feeling" exposure to written letters and numbers. But, if that has not occurred, do not worry! We will help you build that foundation at the same time that we are building the bridge to abstract written language. In fact, a large part of the work done in the early Phases of the Plan consists of this type of tactile activity, done outside the formal homework sessions as part of the "Field Work" that is discussed in

Part 4 of this chapter (perhaps not surprisingly, under the heading "Field Work").

So, the point here – your Action Item -- is that you should (i) read Part 4, entitled "Field Work" and (ii) do a significant amount of Field Work with your child in these early Phases.

2. ***Go Slow.*** As you probably already know, it is very important that you speak slowly and clearly when you are working with your child. It is far better to say too little than to say too much, or to speak too slowly rather than too fast. Approach the material in small chunks, and do not under any circumstance convey a sense of urgency, time pressure, or anticipation of achieving a particular goal. As instructors, we must be like the soft summer wind, or the ancient sage in the old "Kung Fu" television show ("Yes, Grasshopper . . . ."). We are gentle, we are patient, and we are not hurried. We know that time is our ally.

We now proceed to discuss Sample Homework #1 in some detail, introducing a number of Plan techniques. You may wish to read the entire discussion (paragraphs 3 – 10 below) before beginning the first homework session. Of course, it is not expected that you will cover everything discussed in paragraphs 3 – 10 in one homework session! Instead, that material can be covered over multiple homework sessions by (i) using the same Sample Homeworks multiple times, and (ii) deferring some of the material to sessions involving later Sample Homeworks.

NOTE: Many of the ideas covered in our discussion of Sample Homework #1 will also apply to subsequent Sample Homeworks. We will not necessarily repeat all of these ideas in our discussions of those subsequent Sample Homeworks; instead, we will rely upon you to apply these ideas to similar material in subsequent Sample Homeworks.

This is an example of our general approach to discussing the Sample Homeworks: we give reminders and cross-references from time to time, but in general limit the detailed discussion of each Sample Homework to the new material that is contained in that particular Sample Homework. We trust that you will be able to handle "old" material – i.e., material that merely repeats and builds on work done in earlier homework sessions – by continuing to apply the techniques and approaches discussed in connection with the Sample Homeworks in which such material was originally introduced. Do not worry – this may sound challenging, but in practice, as the homework sessions occur over many days, with repeated exposure to the same Sample Homeworks, you will find that you will not need detailed and repeated reminders.

3. ***Breakdown of Sample Homework #1.*** As you can see from the list of objectives above, there are really four concrete skills that are tackled vigorously by the Sample Homeworks in Phase 1:

- Recognition of the child's name;

- Recognition of letters;
- Shape recognition; and
- Counting with one-to-one correspondence.

As we progress through Phase 1, we will encounter these four themes repeatedly. They are discussed in detail in the following four sections.

In addition, there are five abstract concepts that we see bubbling to the surface in Sample Homework #1:

- The idea that things can be thought of in more than one way;
- Pre-reading skills;
- Concepts of inclusion, boundary, etc.;
- Introduction to idea of symmetry; and
- The definition of mathematical objects by lists of characteristics.

These abstract concepts will come up from time to time, opportunistically, throughout the various Phases, and are discussed, opportunistically, throughout this Chapter.

4. *Sample Homework #1:* *Recognition of the Child's Name.* Begin Sample Homework #1 by writing the letters of your child's name, one by one, where indicated at the top of the page. Discuss each letter as you write it, naming it, tracing it, and allowing your child to mark it, all as discussed below. Be sure to emphasize that this group of letters spells your child's name. If your child's name is long and complex -- Guinevere or Oedipus, for example -- you may wish to freelance a bit; perhaps it may be time for a nickname.

- *Discussion Technique: Use Questions Rather Than Statements.* In general, when discussing something with a child in a homework session it is preferable to ask questions rather than make statements. Even if the child does not know the answer, a question involves her in the topic in the way that a "teaching" statement cannot achieve. And, of course, a question can have essentially the same informational content as a teaching statement –e.g., compare "Is this an 'O'?" with "This is an 'O'."

" . . . when discussing something with a child in a homework session it is preferable to ask questions rather than make statements."

- *The Marking Technique.* When you are discussing an object such as a letter, shape or number, allow the child to color the item, or circle it, or mark it, or even cross it out or obliterate it. This is very good reinforcement, because it lengthens the time the child is thinking about the letter, shape or number, while at the same time helping to build a bridge from the concrete to the abstract.

• *The Tracing Technique.* Help the child trace the letters of her name with her fingers, saying the name of the letter. Often, this does not work well — young children are too active and imprecise. But give it a gentle, non-threatening try. Here, we are using Field Work (tactile) techniques to help build that bridge we discussed above. Of course, tracing can also be used with letters, shapes, numbers, etc.

NOTE: You may wish to refer to the Quick Start Module (Appendix B) for a short list of instructions for using Sample Homework ## 1-10

5. *Sample Homework #1:* *Recognition of First Letters: B, O and M.* Next, introduce and discuss the letters B, O, and M one by one.

• *Encourage Both Recognition and Production*. It is a good idea to mix in two different kinds of questions: recognition questions ("Where is the B?"), requiring the child to recognize a letter, shape, number, etc. when it is named; and production questions ("What letter is this?"), requiring the child to produce the name from her memory. Recognition and production are two distinct, albeit complementary, skills, and we want to allow the child the opportunity to approach the new material from different perspectives. Recall that one of the Plan's main objectives is to come at things from many angles in order to allow the child to use her favorite learning strategies in mastering the material.

• *Make Mistakes.* Once your child has begun to identify a letter or shape, make a mistake that she can correct. When you are looking at a "B" ask, "Isn't this an O?" Your child will love correcting your mistake, and more importantly, she will remember the correct answer vividly.

• *How We Choose the Particular Letters to Teach to the Child.* We are somewhat careful in our choice of the first letters to be presented in the homework sessions, preferably using capital letters in the child's name, but most importantly using capital letters that are easy to recognize and distinguish. We usually start with capital B, O and M as in the Sample Homeworks, although we feel free to substitute appropriate letters from the child's name. Letters in common street signs (STOP) are also good. As you will see as you work through the first few Phases, we begin with capital letters, but quickly introduce a few lower-case letters. Throughout this process, we focus like a laser beam on the goal of reading, so we choose letters that will allow early work in sounding out and reading words -- e.g., the "at" cluster of letters (upper and lower case A, T, C, F, H, P, B, R, S), which allow us to work on numerous "at" words such as cat, rat, hat, etc.

For a child named "Sam," the first groups of letters might be introduced in the following order, as they are in the Sample Homeworks:

B, O, M
S
A
T
E (at the end of Phase 1)
C
D
a
e, W, R
P
N, L
i
U
Y, I, V
c (at end of Phase 2)

After many inquiries, allow me to save you some time that could be better devoted to the homework sessions: there is NO hidden message contained in the above letters!

6. Sample Homework #1: *Introducing Shapes.* Ask your child if she knows the name of the shapes at the bottom of the page. Our goals are two-fold here, one concrete and one abstract. First, at the concrete level we want her to learn that these shapes are called circles. Second, at a more abstract level we want to lay some groundwork for the idea that things can have more than one meaning by bringing up the idea that the shape can also be thought of as a letter ("O"). Later, when numbers have been introduced, we can add the idea of zero to the list of different perspectives on this symbol. As illustrated in the sample transcript in paragraph 10 below, we do not push this idea didactically (i.e., in lecture form), instead, we lay the groundwork for the child to learn this idea for herself by just mentioning the idea and perhaps having a gentle discussion about it.

- *Variations On a Theme: Symmetry.* For fun, in a later session try turning the Sample Homework sheet upside down or on its side. Discuss which letters and shapes look different (B, M). Which look the same (O, circle)? Ask your child why the "O" and the circle do not look any different when the paper is turned upside down or on its side. Do not try to teach the answer to this question; we are simply throwing out into the air a concrete example of the deep abstract idea of symmetry. Deep down, your child's incredible inductive learning machine is already beginning to grind away . . . .

7. *Sample Homework #1: Counting.* Now that we know what the shapes are called ("circles," for the educational psychologists who are reading this book), we count them with the child.

• *Counting Is Actually Two Skills*. The idea of counting actually involves two important subskills: (i) memorizing the sequence of numbers ("one, two, three, four . . ."); and (ii) the idea that mathematicians call "one-to-one correspondence," the idea of saying a separate number for each object that is being counted, with no extra numbers between objects and no objects that do not have a number.

• *Memorization Is Part of the Field Work.* The memorization part of counting can be taught in the traditional way, by reciting the numbers in order, often to music, as in the teaching of the alphabet by the Alphabet Song. This can be thought of as being a part of the Field Work discussed below.

• *Touch As We Count*. Here in the homework session, we reinforce the memorization work and develop one-to-one correspondence skills by presenting the child with a row of circles and teaching her to touch them one by one as she counts. Of course, we can supplement this by counting the B's or O's on the homework sheet as well. One way to encourage the necessary one-to-one correspondence is to ask the child to put a dot in each circle with her crayon as she counts it.

• *Future Uses of the Counting Exercises.* We can use these counting exercises for many other purposes: (i) reinforcing a letter that is difficult for the child by counting repeated instances of the letter; (ii) teaching shapes; and (iii) teaching the child to distinguish letters, shapes, numbers, etc. by writing a row containing, say, triangles, circles and squares and asking the child to count the triangles. Another technique for teaching counting is to draw an interesting picture (e.g., of a dog or a shark) and put rows of shapes inside it. To count the shapes, the child must master the idea of a regular approach, preferably left to right and top to bottom. All of these techniques will be utilized in later Sample Homeworks.

8. ***Sample Homework #1:*** *Grade.* At the end of each homework, we give a grade. At these very early stages, the grade can be a simple "√+" (check-plus). Later, we will add more grades (A+, 100%, etc.), not as a measurement or evaluation technique, but as a learning opportunity -- e.g., an early introduction to the idea of "percentage." The grade should always be a good one, because these are "homeworks," not tests; our basic approach is to give the child an opportunity to solve a problem, and then help as needed, so that the experience is always one of success.

9. ***Phase 1 Sample Homeworks In General: Miscellaneous Ideas.***

• *Scribbling*. Do not worry if your child scribbles all over the homework after she finishes, or even as she is doing it – she is just making it her own. In fact, it is not a bad idea to ask her, after you have given her the grade:

> "Sammy, now that you have finished the WHOLE HOMEWORK, would you like to color on it or draw on it as a special treat for a JOB WELL DONE?"

See Figure 10.3 for an example of typical scribbling on a homework sheet.

• *Field Work.* Remember to do Field Work – lots of it -- every day. Field Work is described in detail at the end of this Chapter.

• *Duration of Session.* The sessions should end before the child gets bored. Typically these early sessions last only a few minutes – fifteen minutes would generally be too long. But the key benchmark of length is the child's continuing interest.

"Remember to do Field Work – lots of it -- every day."

• *Relax.* This should be a fun activity, parent and child, no pressure, no disappointment, no hurry. Think of it as the same kind of activity as singing "Eensy-Weensy Spider" or playing peek-a-boo. We are operating on the principle of accretion, rather than catastrophic change. Our math and reading program is not intended to produce sudden large changes in your child's knowledge, but instead to achieve great results through consistent and steady small changes – think of the Grand Canyon rather than Mount St. Helens.

• *Where is the Math?* After all of our talk about the Pittsburgh Plan's innovative approach to math, you may be wondering where the numbers are in these early homeworks! Well, we are nothing if not pragmatists. Reading is more important than math; it is ubiquitous and more financially rewarding. So, we start with reading!

• *Allow Rather Than Force.* We are not really "leading" or "causing" the child's mind to accelerate through the learning process. We are simply providing opportunities that your child will seize when she is ready. Let me restate this, because it is very important: We are not forcing anything at all to happen. We are presenting your child with the opportunity to take steps toward reading and math achievement without pressure, anxiety, or time pressure. We simply allow your child to absorb the concepts when she is ready. Thus, we do not chart progress or worry if we are stuck on the same materials for weeks or months.

Why does this relaxed, no-pressure approach produce such dramatic acceleration of learning in children of all ability levels? It seems too easy, does it not? The reason is very simple – as a society, we vastly underestimate our children's capabilities at early ages, especially in the area of mathematics. Our program simply removes barriers to progress, which then can occur at the child's natural, built-in rate. We enlist as our allies the natural curiosity and the genius for learning that children bring to the world. We are the child's partners, not her drill sergeants.

Sample Transcript: First Homework Session.

Here is one way you could approach Sample Homework #1 (of course, you should feel absolutely free to freelance as you wish; this is just an example).

You could say, "Sammy . . . ."

Right here, we need to make a side comment. Believe it or not, in trial runs of this chapter, several parents mentioned that this approach would confuse their children, because they were not named "Sammy." This is one example of a place where freelancing would be appropriate!

Anyway . . . . just as an example, you could tackle Sample Homework #1 by saying:

> *"Sammy, now that you are almost TWO YEARS OLD!!! -- now that you are SO BIG!!!! – we are going to start BIG GIRL homeworks!!" (Hand her a crayon or pencil; use a pencil only if she is safe with a sharp, pointed object.) "Here is your crayon. Do you like this red crayon? You will use this red crayon for your BIG GIRL homework!!" (Big hug, high five, etc.)*

". . . always pause after asking a question and pay attention to the child . . . "

> *"Let's start by writing your name here on the line at the top. Because this is the line for your name, and it is YOUR homework! Do you know the letters of your name?" (Pause for a minute and listen to her answer – always pause after asking a question and pay attention to the child; the whole point of asking a question is to shift the focus from you to the child and engage her interest and attention.) "Here is your name . . S . . . A . . .M . . . Sam!" (Obviously, you are writing the letters, unless Sammy is extremely advanced!) "Do you want to use your crayon to color your name?" (She can color her name, or circle the letters, or mark the letters, or do nothing at all – anything is OK.)*

> *"Now, let's trace your name!" (Although this technique often fails with restless young children, go ahead and give it a try. She can try to trace the name with her finger or her crayon, whichever she prefers.)*

> *"Now, we are going to start to learn some letters, OK? Don't worry, we are just*

> *starting to learn the letters. It will take a while to learn them! But let's start." (Point to the B.) "Is this a B?" (Pause and listen.) "Yes! This is a B – can you say 'B'? Can you touch the B? Good! Can you touch the B again? Good!" (Point to the B again.) "Is this an O? No!!! That's right, this is NOT an O, it is a B. Do you want to color the B?" (Let her mark, or circle, or color the B.) "Now let's trace the B." (It is OK either way, whatever she wants to do.)*
>
> *"Now, where is the B?" (Help her if she has trouble.) "Do you see another B on the next line?" (Run her finger along the second line.) "Now let's trace the B!" (Take her finger and help her trace the shape of the B. Do this several times.)*

Point back to the B on the first line. Ask, "What letter is this?" (Note that after beginning with recognition questions -- "Where is the B" -- we have switched to a production question, requiring the child to produce the name of the letter from her memory.)

Go through the six letters (two B's, three O's and an M) relatively quickly. It is absolutely OK that Sammy does not have a clue about these letters! This is just step one. After the last letter, you might continue:

> *"Well done, Sammy! We have finished the letters. This was excellent letter work for your VERY FIRST homework! Now we are going to count some shapes. Do you know what these shapes [i.e., the circles] are called?"*

Be ready, it is just possible that your child will answer, "O?" That would be exciting! First, it would mean that she has learned the letter O. More importantly, it would give us a natural opportunity to expose her to the idea that things can be thought of in different ways, or from different perspectives – a rare moment in the Plan when we actually teach, because the child is primed and ready for a new idea. If she does say "O," praise her, give her a high-five, and then say:

> *"You are right! Where we are thinking about letters, this shape is called an O. Very good! But here is a fun thing – sometimes we think about this as a shape, instead of a letter. And when we are thinking about it as a shape, we call it a 'circle.' It is a different name for the same thing – just like sometimes we call you 'Sam' and sometimes we call you 'Sammy.' Just like you have more than one name, so does this shape. It is the letter 'O' and it is also a circle."*

Here, we are seizing the opportunity to introduce one concrete instance of the important general idea that we can think of things in more than one way – one of our key learning objectives.

More likely, when you ask, "Do you know what these shapes [i.e., the circles] are called?"

your child will shake her head or sit quietly. That is fine! Just continue:

> *"This looks like an 'O,' doesn't it? And it is! But now we are thinking about its shape, and its shape is a circle. Can you say 'circle'? Good! Can you point to a circle?" (Help move her hand if necessary.) "Good! Can you point to another circle?" (Help again if needed.) "Do you like circles? Good! Let's trace one with your finger." (Help her trace the circle – let her use her crayon if she wants.)*

Note that in this case, where the child does not herself notice that the shape is an O, we merely mention in passing the idea that the shape can also be thought of as a letter, and then move on. We do not "teach" the idea of multiple ways of thinking about something didactically, we just make a brief mention of it and leave room for the child to uncover this idea for herself, inductively, through repeated future exposures.

Continuing on, point to one of the circles:

> *"Is this a square?" NOOOO, of course not!! It is not a square, it is a .... that's right, a CIRCLE! Is it also a letter? Is it also an 'O'? YES! Very good!"*
>
> *"Now, are you ready to count the circles? We will count them and touch them at the same time."*

If she is a counter, let her try. If she has trouble, or is not yet a counter, take her hand, help her extend the index finger, and count the circles as you touch them: "One . . . Two . . . Three! Good!" Note that we are working here on the very important idea of one-to-one correspondence – touching a circle every time that we count it.

After completing the counting, we finish up:

> *"Well done, Sammy, that was excellent! Now it is time for your grade. We will give you a √+. Here is the √, and here is the plus."*

This brings closure to the session and also lays the groundwork for using the plus sign in addition problems a little later on.

10. The Rest of Phase 1: Sample Homeworks # #2 -14. Now that you have an understanding of the basic Phase 1 objectives and techniques, the remaining Phase 1 Sample Homeworks are merely variations on a theme.

- *Use Techniques and Approaches In Connection With Many Sample Homeworks*. The above discussion of Sample Homework #1 will be germane to many of the following Sample Homeworks. Thus, we will not provide a detailed discussion of each Sample Homework; instead, we will merely comment when a Sample Homework contains something new or when it is time to introduce a new approach or concept. For example, Sample Homeworks ##2 and 4 are merely reconfigured versions of Sample Homeworks ##1 and 3, respectively, and do not require separate discussion.

• *Repeating Homeworks Is OK, As Is Jumping Ahead*. Remember that you should feel free to repeat Sample Homeworks several times in a row, or go back and redo a prior Sample Homework. Likewise, feel free to skip homeworks and jump ahead.

• *Sample Homework #3*. In this homework, we introduce the fourth letter, capital "S," and ask the child to count to six (circles). The techniques described above for Sample Homework #1 will work well in guiding your child through these two extensions of her prior work.

• *Sample Homework #5*. Now we add the letter "A" and introduce a new shape – the square. Ask the child to count how many sides there are on a square. For extra credit, ask her how many sides there are on a circle! This is just a way to provoke thought; discuss it with her but do not try to teach an answer to this "silly" question.

• *Sample Homework #7* -- Repetition. We add the date at the top, next to the name. This is not a very important task from our perspective; mostly, it is just nice to have a date on the homework for reference purposes. Although we do not introduce any new letters in this homework, note that we have worked our way up to four rows of four letters. Also, we are using repetition (three B's in the last row) as a way of cementing the child's knowledge of that letter. Repetition plays an essential role in fueling a child's innate inductive learning mechanism (the innate language ability), as any parent who has read <u>Goodnight Moon</u> a thousand times will attest. Figure 10.3 below shows the actual homework sheet done by Sammy – note the marking of each letter and the post-homework general scribbling all over the sheet. Also note that it is often more fun for the child if the parent copies the Sample Homework in the parent's own handwriting – it seems to give the homework session more of a collaborative feel. Perhaps the most important lesson of Figure 10.3 is that you should feel free to modify the Sample Homeworks to fit your child's particular needs or desires. In the case of Figure 10.3, the counting task (five squares) in Sample Homework #7 is omitted and a picture of a cat is inserted in its stead -- something that was done at the request of Sammy. Sammy did not miss out on the square-counting task; it was included in other sessions based on Sample Homework #7.

• *Sample Homework #8*. We introduce an amateurish picture of a snake containing circles to be counted. Apart from spicing up the counting exercise a bit, this allows the introduction of important concepts such as "inside," "contained in," and "boundary." Again, we do

FIGURE 10.3: One of several actual homework sheets done by Sammy based on Sample Homework #7. Note that the homework sheet was hand-copied from the typed form of Sample Homework #7 in order to enhance the "mutual effort" feeling of the homework session. The counting task (five squares) was omitted from this version of Sample Homework #7 and a picture of a cat was inserted instead, at Sammy's request. The counting task was included in other sessions based on Sample Homework #7. Sammy marked each letter as she read it, received a grade of "✓+," and scribbled on the homework sheet after she was finished. A careful look at the left side of the homework sheet, slightly below center, reveals a drawing of a winged horse presumably done by one of Sammy's older siblings.

not teach these ideas in a formal, lecture-style way. Instead, we simply discuss with the child the idea that the circles are inside the snake, that the snake is like a fence around the circles holding them inside, and perhaps we even use the word "boundary." [2]

We are not teaching these important abstract concepts; we are merely discussing one concrete instance of them, helping the child's induction mechanism ferret out the abstract concepts over time.

For many people, it is heresy even to think about introducing this type of abstract concept – "inside," "contained in," and "boundary" -- as part of the education of young children. They will point to many studies, and great thinkers beginning with Piaget, each confirming that young children cannot handle such abstraction. We have two responses:

(a) First, and most importantly, we are not trying to teach these concepts to our very young children. Such an effort would undoubtedly fail. Instead, we are

2 Children learn the words they hear. If we use words such as "owie" and "boo-boo," they will learn those words. If we use words such as "cut" and "bruise," they will learn those words instead. So, we might as well use good words! This is just one more example of the ways in which parents limit their children by their limited expectations of them.

making sure that these rich concepts are in the child's intellectual environment. We are making sure that we do not limit the child by limiting the ideas to which she is exposed. We are allowing her the opportunity to bring her formidable intellectual weaponry to bear on these important ideas over time.

(b) Our second response is less considered: Look at the state of our schools, look at the results obtained by these critics – why in the world should we listen to them? Perhaps the time has come to deep-six these corduroy-patch-on-the-elbows bozos and trust our children for a change.

"Perhaps the time has come to deep-six these corduroy-patch-on-the-elbows bozos and trust our children for a change."

• *Sample Homework #9*. This is an important homework, in which we introduce several new things:

(a) *The "AT" Word Cluster.* The capital letter "T" makes its first appearance.[3] This is significant, because the child's "tool set" now includes both A and T, enabling us to begin working on reading the "AT" family of words – at, bat, cat, fat, hat, in, pat, rat, and sat.[4] We will do this by beginning to discuss the sounds that letters make – probably not quite yet (unless you want to), but soon.

(b) *New Shapes: Triangles and Rectangles.* There are two new shapes in this homework: triangles and rectangles. We can exploit this rich new material in a number of ways. (By the way, please do not feel that you have to cover every angle on every topic in a Sample Homework in each homework session based on that Sample Homework! Just cover them all over time.)

(i) *Distinguishing Shapes.* First, of course, have your child name the shapes (introducing them in the same fashion that circles were introduced in Sample Homework #1). Discuss how they can be distinguished from each other, how they are different. Focus on the number of sides they have.

Count the sides. For fun, ask again how many sides a circle has, then discuss this idea (one side, not really any sides, can you have a side that is not a straight line, etc.; anything is really OK, we are just looking for discussion.)

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3 As a reminder, you should use the techniques described in connection with Sample Homework #1 to introduce your child to this new letter.

4 We included the word "in" just to see if you were paying attention – were you?

Ask the child to color each triangle blue, color each circle red, etc. This can be used not only to reinforce the names of the shapes, but also to reinforce the child's knowledge of colors.

(ii) Squares and Rectangles: Deep Idea of Inclusion; Definitional Lists of Characteristics. Here we try to plant a few seeds for much later harvesting. Specifically, we ask a number of leading questions and then discuss them.

Answers are provided in parentheses for your benefit, but you should not "teach" the answers to your child. Just discuss the questions and throw out some ideas, guided by the parenthetical answers and by your own knowledge and instinct.

- Ask your child to count the sides on the rectangle, then count the sides on the square. (They both have four sides!)

- Ask if the corners all look the same (they do; they are all square).

- So how are the square and the rectangle like each other? (four sides, square corners.)

- How is the square different from the rectangle? (The square has sides that are all the same length; the rectangle can have a width that is different than its height.)

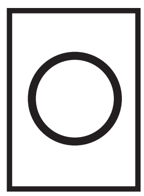
- Can we think of the square as just being one kind of a rectangle – a special kind of rectangle, with its own name, because all of its sides are the same? (Again, we do not expect the child to understand this, and we do not really "teach" it, we just float it out there, eliciting as much of it as possible from the child herself. We simply want to get the idea out there into the environment.)

With these questions, we are working toward an understanding of a specific instance of the deep mathematical concept of <u>inclusion</u> – the idea that squares are a subset of rectangles. Also, we begin to focus, through a specific example, on the important abstract idea that mathematical concepts can be defined in terms of a list of properties (e.g., a rectangle has four sides and four square angles).

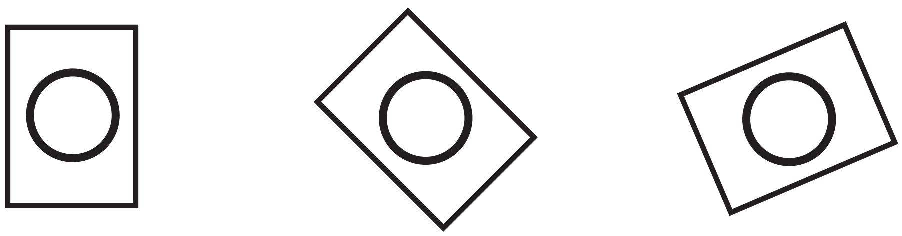
(iii) *Introduction to the Deep Idea of Symmetry.* In the universe of ideas, symmetry is the royal flush, the top dog, the latte grande; it is the blue-ribbon, black-belt, Achilles, Alexander, Lancelot and Roland of ideas.[5] Symmetry defines beauty in art and in love; it separates a great poem from a lesser one; it even underlies the great advances in theoretical physics over the last century. By now you know that the Plan does not shy away from these types of grand-slam ideas, but instead embraces them. So it will not surprise you to hear that we tackle this idea – at least, one concrete realization of it – in our very first Phase!

Here is how we do it, spread out over many homework sessions (this is a long project, but that is OK; to paraphrase Arlo Guthrie, "We are not proud, or tired"):

- Using a computer, print out a page that is blank except for a large circle in the middle of the page. Our goal here is to have a perfect circle in the middle of a blank page, like this:



- Now, have your child rotate the page a little bit, then a little more, and so on:

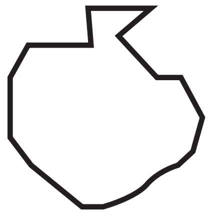


In each case discuss whether the circle looks different after it has been rotated. For example, ask if the circle is "pointing in a different direction." Make sure to distinguish between the direction of the rectangular page and the appearance of the circle. If this is confusing your child, cut the circle out carefully with scissors and just use the circle. (Of course, the circle looks the same after it is rotated because it is highly symmetric.) Do not teach, just discuss!

- Now draw a circle by hand on a separate piece of paper, making sure

5 Roland? Yes, Roland! And Oliver too, for that matter. See <u>The Song of Roland</u> (Anon. c. 1050?). We mention Roland for his iconic status as the greatest and bravest of Charlemagne's knights; we do not in any way view the Crusades with favor.

that it is noticeably irregular, like this:



Do not be proud; make sure that it is obviously irregular. Ask your child about this crummy new circle: "This is not really a good circle, is it? Why not?"

The answer, of course, is that the circle does not look the same all the way around; some parts look a little different.

• Now have your child rotate this new crummy circle. Ask her if it looks different after it has been rotated, if it points in a different direction. Again, make sure that she is not focusing on the orientation of the rectangular page; if she is, carefully cut out the flawed circle with scissors and just rotate it.[6]

• Try another idea with each of the circles – have your child close her eyes while you rotate the paper so that the top is now the bottom (so the rectangular shape of the page does not give away the rotation). When she opens her eyes, can she tell if the perfect circle has been rotated? How about the flawed circle? Switch roles and let your child turn the pages while you cover your eyes. Let her ask you if you can tell if the circles have been rotated.

Of course, unlike the perfect circle, the flawed circle looks different after it has been rotated, and your child probably will be able to tell that it has been rotated when she opens her eyes. Why? Because the flawed circle is not the same everywhere; it lacks the rotational symmetry of a perfect circle.

• Do the same thing with other shapes: a square; a rectangle; and finally an equilateral triangle (i.e., a highly regular triangle having three sides of equal length). These shapes do look different unless you rotate them certain special amounts (e.g., multiples of 90 degrees for a square, multiples of 120

6 Why not just cut out the circles to begin with, eliminating the potential confusion caused by the orientation of the rectangular page? Because we are not teaching, we are exploring and learning, and the circle on the page is a richer learning environment, with more interesting symmetry aspects, than the circle by itself.

degrees for an equilateral triangle.) Discuss.

- Do the same thing with an irregular shape – one that never looks the same unless you rotate it completely around 360 degrees. (This is really no different than the flawed circle from above.) Discuss this with the child. Ask her to comment: "When does this look the same? Can we tell if the top has moved to a different spot?" Etc.

- Again, do not try to teach your child the idea of symmetry, just discuss the circles and other shapes with her. We are merely enriching her environment so that she has the opportunity to develop – over a period measured in years! -- a deep and intuitive understanding of this important concept.

• *Sample Homework #13*. Here we add the letter "E." Also, we begin to standardize the first few letters of the letter recognition task, so that they are the same for most homeworks. There are two ideas behind this standardization. First, it allows the child to start every homework with a successful streak. Second, it gives us another angle into the beginning of reading. We choose the standardized beginning letters so that they form a simple, familiar word, which can then be used as an early reading task (eventually, once letter sounds have been introduced). We will call this word the "Key Word." In this case, the Key Word is "S A M," chosen, obviously, because the student is named "Sam." Henceforth, at least for a while, most homeworks will begin with the Key Word.

PART 3: Detailed Discussion of Phase 2: The Early Consolidation Phase

Sample Homeworks: Sample Homeworks ## 15 – 33.

Objectives: Phase 2 is a consolidation period in which the child builds a base for the two great and intrepid adventures that will begin in Phase 3, namely, reading and elementary math operations (addition and subtraction). As with Phase 1, it is possible that your child will not yet be ready for a full-scale schedule of homework sessions, and once again Field Work is heavily emphasized in this Phase.

Here are the objectives of Phase 2:

- Continue to reinforce the letters and shapes from Phase 1;
- Continue to do Field Work regularly;
- Count higher (to 27);
- Introduce backward (countdown) counting in the "Blast-Off" game at the end of each session (this lays the groundwork for subtraction, which will come along almost before you know it!);

- Introduce the first numbers: 1, 2, 3, 4;
- Introduce many new letters, including ones that are important for reading readiness (vowels and letters contributing to the "at" cluster of words). In the case of the Sample Homeworks, the new letters are C, D, a, W, e, R, P, N, L, i, U, o, V, I, Y and c;
- Begin introducing the letter sounds, especially for letters that are relevant to early reading work (a, A, S, B, C, T);
- Teach recognition (and possibly sounding out) of the Key Word (e.g., "SAM" in the Sample Homeworks);
- Continue exposing the child to concrete instances of the important general concepts of Phase 1: included/bounded; definition by list of characteristics; symmetry; nested (hier archical) sets; and thinking of things in more than one way; and
- Introduce the idea of vowels as a special category of letter.

Time Frame for Phase 2. Field Work continued to be more important for Sammy than a full schedule of homework sessions. As a result, Sammy took approximately eight months to finish Phase 2, starting at age 29 months and finishing at age 37 months. You can see that she continued to work at a relaxed pace – nineteen homeworks, each repeated on average three times over 34 weeks, working out to between one and two homework sessions per week -- supplemented with a great deal of Field Work.

Detailed Suggestions and Comments for Phase 2:

1. *Continuation of Phase 1 Approaches.* You should continue to follow the good practices discussed above with respect to Phase 1. For convenience (and because repetition works well with adults too), we will list these suggested approaches again:

- Do Field Work;
- Go slow;
- Do not try to cover everything in each session; just cover everything over a period of time;
- Use questions rather than statements;
- Encourage your child to circle, cross out, or otherwise mark letters, numbers, etc. that are being discussed;
- Encourage tracing (finger or crayon);
- Elicit both recognition and production;
- Allow your child to correct your (intentional) mistakes;
- Touch as you count (one-to-one correspondence);
- Repeat homeworks;
- Repeat material within a homework session;
- Occasionally discuss distinctions between shapes – how one tells them apart; how they are different;
- Occasionally "float" the idea that squares are one kind of rectangle, but rectangles are not one kind of square; and

- Occasionally raise the symmetry notions discussed in Part 2 (using the techniques discussed in Part 2).

2. The "Standard Discussion" -- Be Honest About the Plan's Approach to New Ideas. No matter how hard you try to keep things relaxed, there is always the possibility that your child will be on guard, looking for signals that you expect more of her or are unhappy with her performance. This can create problems in the many cases where we introduce new concepts at an early stage. From our perspective, we are just launching these new ideas into the cognitive ocean, knowing that they will not find land for a while (possibly a great while!), but at least hoping to enrich the environment of ideas in which the child's powerful inductive learning apparatus is operating.

> **" . . . we follow the theory of the Plan; we trust the child, and make her our partner."**

The problem is that your child may have a different perspective; she may worry that she is not grasping some of the concepts you are introducing, and may become anxious or feel pressure. How do we handle this? Quite simply – we follow the theory of the Plan; we trust the child, and make her our partner. We tell her repeatedly, especially when we are introducing a new idea, that our whole idea is to learn ideas over time rather than the first time they show up.

Let me give you an example. You might preface a discussion of squares being a kind of rectangle by saying something like this, slowly, and in very relaxed, happy way:

Sample Transcript: New Ideas

> *"Sammy, now we are going to talk about these shapes a little bit. We will talk about some new ideas. These new ideas are fun, but they are very hard. These are SIXTH GRADE IDEAS! Even fifth graders don't know these ideas! We do NOT expect you to know these ideas yet! Because you will not be in sixth grade until next year!" [Allow her to correct this obvious mistake.] "We are just going to talk about these ideas for a minute for fun. You know how we do things, right? We talk about ideas fifty times! And of course you do not understand them the first time, or the second time, or the third time. But after fifty times, suddenly, without even trying, we will find that you understand the idea! How about that! Isn't that neat? So this is just the first time, just for fun. Will you understand it today? Of course not! That is OK! We are just having fun for a minute talking about SIXTH GRADE IDEAS!"*

You should tailor this kind of discussion to fit your child's personality and attention span, but maintain the basic approach of relieving your child's potential anxiety by making her your partner rather than your student.

3. ***The Phase 2 Homeworks: Sample Homeworks ##15-33.*** We discuss below those Sample Homeworks in which new ideas or techniques come up. Of course, you should use these ideas and techniques in other Sample Homeworks as well, where appropriate. For example, the ideas discussed below in connection with the cat diagram of Sample Homework #15 work equally well with the strange animal diagram in Sample Homework # 16.

- ***Sample Homework #15.***

  o *Key Word.* Notice that the first row of letters begins with the simple Key Word that we have decided to emphasize, in this case the child's name, "SAM." See if your child notices that the first few letters are her name or the other Key Word you have been showing her in past homeworks (and discussing as part of the Field Work). If she does not notice that fact, prompt her gently.

  o *Letter Sounds In the Key Word.* Work with her gently and briefly on the sound that each of the letters in the Key Word makes (which again also should be part of the Field Work). Show her briefly how these letter sounds can be said close together, resulting in the saying of the Key Word itself. <u>Do not expect that she will be able to reproduce or even understand this idea</u>. It could be many months before she makes the cognitive leap from knowing the sounds of individual letters to understanding how those sounds can be merged into the sound of a word. Be patient; we are just laying the groundwork now.

  o Letter Sounds in the "at" Cluster of Words. Note that we increase the counting to 14 and add the new letter "C," which will be useful for early reading work (e.g., in the word "CAT"). Begin teaching the sounds made by the "at" cluster of letters, i.e., the letters that can be used in simple "at" words such as "cat," "fat," etc. So far, we have introduced a, A, T, c, B, and S from this cluster. This early phonetic work should be done both in the homework sessions and as part of the Field Work. Note that our Key Word, "Sam," has the same vowel sound as the "at" cluster of words. <u>Teaching the sounds of the "at" cluster of letters should now become a regular part of most homework sessions</u>.

  o The Cat Counting Diagram. You may wish to use the cat diagram at the bottom of the page to discuss the concepts of inclusion/boundary:

Sample Transcript: Incusion/Boundary

> *"Sammy, can you see the big circle that makes the cat's body? Can you trace it with your finger? Are the little circles inside the big circle? Or are they outside the big circle? YES! Very good, the little circles are INSIDE the big circle. So then, is that big circle a 'boundary' around the little circles? A boundary is like a fence.*

Is the big circle a boundary around the little circles because they are inside the big circle? Can you say 'boundary'?"

Also, it might be fun to ask your child to spot circles (the cat's body, head, feet and eyes) and triangles (ears). If you desire, you can slowly write out the word "cat" next to the diagram and discuss the letter sounds, etc. with your child. And, of course, your child will probably enjoy coloring the cat! (Remember, we generally use the Sample Homeworks more than once, so do not feel pressured to get this all done in one session!)

"Can you say 'boundary' ?"

o *Grades.* In Phase 1, the grade given at the end of every homework session was a "√+" unless you free-lanced a bit – which is, of course, perfectly OK! Now we expand the grades; we still give a √+ at the end of every session, but we also either select a letter or (preferably) allow the child to select a letter, and give that grade as well (usually also with a plus). So, if the child selects "M" as her letter grade, the grade would be "√+, M+." Also, we give a grade of the form "VE__", where the VE stands for "Very Excellent" and the child selects the part of the homework that she felt best about – for example, the grade could be "VENW" for "Very Excellent Number Work." Note that we give grades that teach!

• ***Sample Homework #18.*** Here, we introduce the new letter "D" – now we can add the word "DAD" to our list of words to be sounded out in the homework sessions and in the Field Work. If you want, you could also add the word "MOM" to that list, but we have always been concerned that it might be best to limit yourself to one vowel sound at first (the "a" sound in "Sam," "cat" and "DAD").[7] Feel free to make your own decision; freelance a bit!

• ***Sample Homework #19***. There is a lot going on in this Sample Homework:

o *Lowercase "a."* We introduce our first lowercase letter, "a." Obviously, this is helpful for Field Work in connection with the "at" cluster of words, since most occurrences of those words (cat, sat, bat, etc.) out in the "field" will use lower case letters. Surprisingly, lowercase "t" does not show up until Sample Homework # 34; this is an oversight in this particular set of Sample Homeworks and should probably be addressed by bringing it up earlier either in the homework sessions or in the Field Work. (You will be amazed at the ease with which your child can overcome this kind of oversight!) Note our use of repetition – the entire bottom row of letters consists of "a" repeated five times.

o *Lowercase "a" Again*. You may wish to supplement the Sample Homework with a brief introduction to the handwritten small "a" – which looks like an italic typed "*a*."

7 There is no truth to the despicable rumor that I favor early reading of the word "Dad" over the word "Mom" because I am a father.

o *Weird Diagram.* There are many things that can be explored using this diagram – a few of which are set forth at the bottom of the Sample Homework.

- ❑ You can ask your child to count the circles and the triangles.
- ❑ She can then color the circles blue and the triangles red.
- ❑ You can discuss the ideas of boundary and inclusion as usual.

Special Topic: Parallelograms

Just for fun, you can introduce and explore the rich idea of a parallelogram, which is basically a rectangle where the top side has been pushed to the left or the right, leaving the right and left sides slanted but still parallel to each other (i.e., some of the angles are not ninety degrees). More technically, a parallelogram is a four-sided figure where both pairs of opposite sides are parallel to each other.
In the Sample Homework, the front part of the weird animal's body is a parallelogram.

The word "parallelogram" is not really critical, although your child is definitely capable of learning it and it is a great word; it is a much better word than "Voldemort" or "Snicket"! Instead, the real value of this new shape lies in the many deep relational ideas that arise naturally in a discussion of it. I guarantee that your child will not be ready to understand these ideas, but it is probably worth your while to put them into play in a relaxed way over a few different sessions using this same Sample Homework (it would be excessive to try to cover all of this new conceptual material in one session!)

Because we are drifting into areas that are fairly technical, let us remind you again: It does not matter at all if you do not understand these areas, and are not able to answer questions or even know what to teach. None of that matters! What matters is very simple: namely, just the fact that you and your child discuss ideas such as how the body of the weird beast (i.e., the parallelogram) is like a rectangle and how it is unlike a rectangle.

Here are some of the ideas relating to parallelograms that you might raise and discuss with your child in a gentle and gradual way:

- How is a parallelogram like a rectangle? (Four sides, with opposite sides being parallel, meaning that they go in the same direction.) How is a parallelogram not like a rectangle (corners are not square). Here, it might be fun to draw various pairs of lines, some parallel and some not, and discuss the idea of lines being parallel (i.e., "going the same way").

• Discuss the idea that a rectangle is a kind of parallelogram (since the rectangle has opposite sides that are parallel, the key definitional characteristic of a parallelogram.) But a parallelogram is not a rectangle, because a rectangle must have square corners.

• For real fun, discuss the ideas that a square is a kind of rectangle, and a rectangle is a kind of parallelogram. (This is a super-sized example of nested or hierarchical sets.) See if you can lead the discussion into the notion that this means that a square is a kind of a parallelogram.[8]

• An interesting Field Work project relating to this idea is to take a flimsy rectangular box and squash it or push the top a bit to the side so that the rectangle formed by each side of the box becomes a parallelogram.

• ***Sample Homework #20.*** We introduce three new letters: W, e, and R. Probably for the moment only the "R" should be added to the list of letters that you and your child are sounding out. In the diagram, you may wish to discuss how rotten the circles are – they are really "ovals." (Why not teach this word to your child? If she can learn "Sponge Bob Square Pants" she can learn "oval"!) What is the difference between a rotten circle (e.g., an oval) and a good circle? Discuss.

• ***Sample Homework #21.*** The new letter is "P," which again fits nicely into the "at" cluster of words ("pat") and thus should be taught immediately as a sound as well as a letter. Ask your child to compare the "P" with the "R" that she learned in the prior Sample Homework – what are the similarities; what are the differences? In the counting exercise, ask your child to count the triangles, then the rectangles, then the circles. Which is the biggest number?

• ***Sample Homework #23.*** This Sample Homework is notable for the introduction of the session-ending "Blast-off." Simply count backward from ten slowly, as in a countdown, finishing up "3 . . 2 . . 1 . . BLASTOFF," at which point you should do something dramatic, such as lifting your child high into the air or tickling her. Then do the same thing again, only this time counting backward from twenty. These countdown drills lay the necessary groundwork for our early introduction of subtraction, as you will discover in Phase 3. These countdowns should also be practiced (together with regular forward counting) as part of the Field Work.

• ***Sample Homework #24.*** Numbers make their first appearance! As part of your discussion

8 For the mathematically interested reader, this is one example of a deep mathematical concept known as the transitive property of set inclusion.

of each number with your child, you should discuss the idea that numbers tell us "how many" of something there are – e.g., how many pennies, or books, or fingers. In particular, it is good, when discussing "3" for example, to put three pennies down on the table and have your child count them, emphasizing (in our usual low-key style) that the symbol three means the same as the "counting" three. Figure 10.4 shows the actual homework done by Sammy – you can see how excited she was at the introduction of numbers; she did her best to write "1" and "2" next to those numbers on the homework sheet. Also note her efforts to write her name and age at the top of the page.

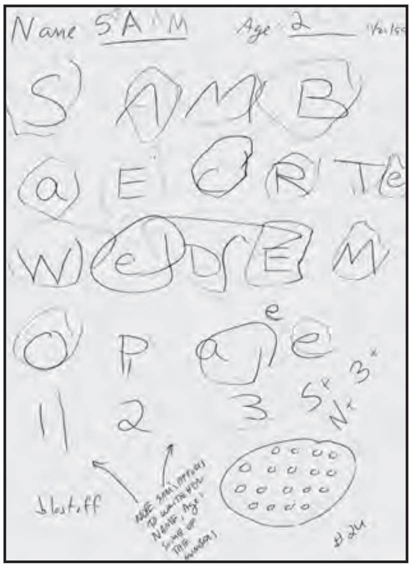


FIGURE 10.4: Note Sammy's efforts to write the letters of her name, her age, and the numbers "1" and "2" at the bottom. Also note that she chose the number "3" as one of her grades, thus receiving a "3+" in addition to an S+ and an N+

• ***Sample Homework #25.*** "L" and "N" are introduced. Since L is a hard sound that is not involved in the "at" words, we often do not teach it phonetically at this point. You should decide for yourself how you want to handle such matters.

• ***Sample Homework #26.*** Lowercase "i" enters the arena. Because we teach new letters by asking our children to look hard at the letters and try to see what features distinguish them, we find that Plan participants often refer to small "i" as the letter "dot!" After years of being bothered by this, we finally hit upon the solution. Now, we encourage the child to think of the "dot" as the letter's "eye" looking at her.

• ***Sample Homework #29.*** As we continue through this consolidation period, we introduce two new vowels: "U" and "o." You will be pleased to know that children have very little difficulty extrapolating from capital "O" to small "o." You may wish to use the diagram in this Sample Homework to introduce ideas related to separation into pieces (concepts that are important in many areas, e.g., division). You might ask, "How many pieces is this shape divided into? How many circles are in each piece?" Where is the middle piece? Can you trace the boundary of the middle piece with your finger? Can you color the three pieces different colors?"

• ***Sample Homework #31.*** Note that we are now counting to twenty-seven. We introduce three new letters – "I," "V" and "Y" – and then launch the concept of vowels. We do a number of things with the idea of vowels:

o We memorize them (a, e, i, o, u and sometimes y). A little chant sometimes helps with this memorization task:

"a, e, i, o, u and sometimes y cha cha cha [tickling your child on the 'cha's]"

For the musically inclined, we sing this chant with "a," "e," "i," and "o" being quarter notes and "u," "and," "some," "times," "y," and all three cha's being eighth notes, making the whole chant two measures long. This chant should be repeated several times. Memorization of vowels and practicing of the chant should also be part of the child's Field Work.

o We use vowels to introduce the idea of consonants. In essence, we try to help the child figure out the simple rule that the consonants are those letters that are not vowels. Then, we do not have to memorize the consonants, we can just memorize the vowels and apply this simple rule to see if a letter is a consonant.[9]

o We discuss the fact that vowels all have sounds that are very easy to make, and give examples (all have the mouth open, lips and tongue relaxed, etc.).

o We discuss the interesting idea – related to inclusion – that "y" is sometimes a vowel and sometimes not, depending on how it sounds. Y has one basic non-vowel or consonant sound (as in "yikes") and two predominant vowel sounds (as in "Sammy" and "sky").[10]

o At some point, often after the sounding out of words has begun, we discuss the fact that English is a very hard language in which every rule is broken somewhere. The many sounds that vowels can have is just one example of this – when we are reading, sometimes we have to

" . . . English is a very hard language in which every rule is broken somewhere."

9 It will probably not surprise you to learn that this is a concrete example of an important idea from set theory. Specifically, if we let L be the set of all letters and V be the set of all vowels, then the set of all consonants C can be expressed as L\V, the "complement of L relative to V."

10 This is true only in English; in other languages, "y" can have different sounds. We should mention here a point that is probably obvious – the Pittsburgh Plan is focused on teaching math and the reading of the English language. We considered at one time developing a program for teaching the reading of Japanese, but were hindered by the fact that none of us could actually speak Japanese. In hindsight, it seems ludicrous, doesn't it, that we would attempt to teach a language without speaking it – but then again, how is that different from the teaching of math in some of our elementary school classrooms?

try different sounds for the vowels to figure out what word we are reading.

o As soon as possible, we allow the child to write the vowels herself.

o Important! We practice the vowel sounds as we study them, proceeding gradually (e.g., starting with "a," then moving to o, e, i, u and y in that order). Always, as we are discussing a vowel's sound, we also discuss possible alternative sounds the same vowel might occasionally make.

• ***Sample Homework #32.*** The number "4" is introduced.

• ***Sample Homework #33.*** Lower-case "c" makes its first appearance – useful for our upcoming work on the word "cat."

PART 4: Field Work.

When you look at the sample homework materials, you will see immediately that the initial homework sessions cannot possibly last as long as fifteen minutes. Do not get too excited -- we still want our entire fifteen minutes! Instead of devoting the whole fifteen minutes to the homework session, we want you to make up the difference (and preferably a lot more) doing "Field Work."

"Aha," you exclaim, "I knew you weren't telling the truth! You promised that the Pittsburgh Plan would only take fifteen minutes a day, four days a week. Now you are asking for even more time!"

We admit it; you caught us; we were sandbagging. We misled you in the hope that by the time you discovered the truth, you would not balk at the idea of spending up to <u>two</u> or even <u>three</u> hours a week to change your child's life forever. And we still hope that you are willing to do just that!

What do we mean by Field Work? We mean reinforcing the ideas of the Sample Homeworks by practicing them now and then, through play and otherwise, and by finding concrete examples of them in day-to-day life. Field Work can begin at any age — there is no need to wait until your child is old enough to commence homework sessions.

Here are the key Field Work tasks for Phases 1 and 2:

• Make it a constant practice to spot letters, shapes, numbers and words with your child in books, on signs, and everywhere else they occur in day-to-day life (even skywriting!).

o For example, if you are working on capital "B" and the butcher stand at the grocery store has a large sign in the meat case saying, "Big Bucks Butcher's Break: Buy Beef Brisket Before Breakfast," grab the opportunity; go wild pointing out B's with your child!

o Note that we say "with your child," not "to your child." This distinction neatly sums up most of the difference between the Plan and elementary school.

- Use tangible toys to reinforce the ideas of letters and numbers.

" . . . we say "with your child," not "to your child." This distinction neatly sums up most of the difference between the Plan and elementary school."

  - For example, buy a set of plastic letters, and then play games with them. Put four or five letters on a stool across the room, and then, while hugging your child, tell her to run across the room, get you a "B" and bring it back. (If you do this, it would be best if one of the four or five letters were a "B.") Or, have a handful of letters, and ask her to select the "B" and take it over and put it on the stool.

  - Play with letter blocks (blocks that have letters on each side), and ask questions about the letters on the blocks, or ask your child to stack the next block so that the "B" is on top.

  - Get plastic letters with magnets inside them (make sure that the magnets cannot be swallowed by your child), and play games involving sticking the letters on the refrigerator and taking them off.

  - Everyone seems to think flash cards are evil, that they are the visible manifestation of overzealous, pressure-packed parenting. Not so! Think of them as a fun game and your child will too. Make up a few flash cards and play games with them (homemade flash cards are much better than store-bought ones, for the same reasons that crummy graphics are better than professional graphics in the Sample Homeworks). If possible, the games should involve your child handling the cards. At this early stage, the cards should simply have the letters, numbers and shapes on them, as well as possibly the child's name and the Key Word (if different than her name). Later, we will suggest possible expanded uses of flash cards.

- Have your child count objects (pennies, blocks, toys, people at the table, etc.), always focusing on one-to-one correspondence.

- Say a number between one and ten (for fun, you can try eleven or twelve sometimes to see what happens) and have your child hold up that many fingers. Then hold up fingers and ask your child to tell you how many you are holding up – strive to reach the point where she recognizes the number and does not have to count. This is important precursor work for the addition and subtraction problems she will encounter in Phase 3.

- Use your time in the car to practice memorization tasks:
  - Counting (reciting the numbers in order – the memorization part of counting)
  - Counting backward (blastoff)

- o Calling out a number and having the child hold up that many fingers
- o Reciting the alphabet and singing the alphabet song
- o Reciting the months in order
- o Reciting the days of the week in order
- o Spelling your child's name and the Key Word
- o Practicing letter sounds ("What does a 't' sound like? What are the different sounds that an 'a' can make?")
- o Reciting the vowels and chanting the vowel chant

• Point out real world examples of symmetry (e.g., twin towers of an apartment building, etc.)

• Discuss real-world examples of set inclusion. For example, "Are shirts a kind of clothes? [YES] Are clothes a kind of shirt?" [NO]

• Read to and with your child at every opportunity. (This is not technically Field Work, because it is not really optional; it is as important to your child's welfare as feeding her!)

• Do the types of things described in paragraph (b)(iii) of the discussion of Sample Homework #9 above using drawings of circles, squares, rectangles and other shapes.

Phase 2 is done, and not a moment too soon! Now it is time to strap on your seat belts, because the real adventure is about to begin.

CHAPTER 11:
Crossing the Rubicon

In 49 B.C., Julius Caesar brought his great Roman army to the Rubicon, a small river in northern Italy. According to legend,[1] he paused for a moment, feeling the burden of centuries of tradition and the weight of ancient Roman law proclaiming it treason for any general to cross the Rubicon with a standing army. Then, turning to his commanders, he proclaimed, "Iacta alea est!" – "The die is cast!" – and marched across the Rubicon, toward Rome and into history.

You and your child are now poised at your own Rubicon. So far, your work under the Plan has been dominated by Field Work and has been traditional in all but its deepest undercurrents. Few educators would criticize the program to date. That is all about to change. In this Chapter, your young child will begin to do addition and subtraction problems before she has learned to recognize numbers beyond four! She will begin to read before she has learned even half of the alphabet! These steps fly in the face of decades of educational tradition in the United States; they are gross violations of the conventional wisdom of the American educational system. And there is no turning back, because once your child begins to learn these skills she will hunger for more. Your child is ready, but are you? Will you cross the Rubicon?

> **" Your child is ready, but are you? Will you cross the Rubicon?"**

Iacta Alea Est!

PART 1: Detailed Discussion of Phase 3: The Rubicon Phase

Sample Homeworks: Sample Homeworks ##34 – 63.

Objectives: Here are the objectives (the monster objectives are in bold type):

- Continue to reinforce the letters, shapes, numbers and vowels from Phases 1 and 2;
- Continue to do Field Work regularly;
- Introduce more numbers: 5, 6, 11, 8, 7, 9, 0, 10;
- Introduce more letters, again including ones that contribute to the "at" cluster of words: w, u, t, m, s, H, v, y, p, b, r, x, G, X, h, F, g, Q, Z, f, z;
- Continue to emphasize letter sounds, especially for letters that are relevant to the "at" cluster of words (a, A, S, B, C, T, t, s, H, p, b, r, h, F, and f);
- **Introduce addition using two approaches: counting objects (e.g., pennies); and using fingers;**
- **Introduce two styles of addition, sideways and up-and-down;**
- Encourage the child to write letters, numbers, her name, etc.;
- Introduce subtraction, again by the same two methods (object counting and fingers);

1 And the historian Suetonius, whose phrasing we prefer over the more common "Alea Iacta Est."

- **"Read" addition and subtraction problems (math as a language);**
- **Read "at" words both phonetically and by recognition;**
- Hope the child is excited enough to begin creating her own homework sheets (and encourage this);
- Continue exposing the child to concrete instances of the important general concepts of Phase 1, especially included/bounded and thinking of things in more than one way; and
- Introduce the important concept of zero as a special number.

Time Frame for Phase 3. Sammy took approximately eight months to finish Phase 3, starting at age 38 months and finishing at age 46 months. Although Field Work continued to be important, the homework session pace was picking up significantly as Sammy's excitement grew about her newfound capabilities. Each homework was done at least twice, and Sammy created a number of her own homework sheets. In all, there were approximately seventy homework sessions over the nine months of Phase 3, which works out to a little less than two sessions per week – a significant increase over the pace of Phases 1 and 2!

Detailed Suggestions and Comments for Phase 3:

1. ***Continuation of Phase 1 and 2 Approaches.*** We list the most relevant of these suggested approaches again for your convenience:

- Do Field Work;
- Go slow;
- Do not try to cover everything in each session;
- Use questions rather than statements;
- Encourage your child to circle, cross out, mark or trace letters, numbers, etc.;
- Elicit both recognition and production;
- Allow your child to correct your (intentional) mistakes; and
- Repeat homeworks and material within a homework session as desired;

2. ***Sample Homework #34.*** We hit the ground running --

• *Writing Name.* We now encourage the child to write her own name in the blank space provided. If that is difficult for her, we write it, with large spaces between the letters, and ask her to try to write each letter next to the one we wrote.

• *New Letters.* We introduce w, u, t, and m – with "t" being especially important due to our emphasis on the "at" cluster of words.

• *Reading.* Note that the word "Sam" appears twice in the rows of letters, and the word "cat" appears as well. First let your child recognize the individual letters; see if she notices that they form a word.

If not, point the words out to her and work on sounding them out. (We discuss the process of "sounding out" in more detail in our discussion of Sample Homework #35 below.)

• *Addition!* We introduce our first mathematical operation, addition! First, we tell the child that we are about to do a very big thing – addition, or pluses. We use those words a few times and have her repeat them, just to get started. Then, we have the Standard Discussion about our approach to new ideas – that we introduce them but do not expect the child to understand them at first, etc. (as set forth in the discussion of Phase 2 in Chapter 10 above). We proceed to do <u>three things</u> with these two addition problems – but remember, although the Rubicon was crossed in a few minutes, Rome was not built in a day:

> **". . . remember, although the Rubicon was crossed in a few minutes, Rome was not built in a day . . ."**

o *<u>Reading the Problem.</u>* You may recall that a significant insight in the theory underlying the Pittsburgh Plan is that math can be learned as a native language. It is probably not surprising, then, that the very first thing we do when we see a math problem is to read it, just as we would read a sentence. The child already knows all of the symbols in the two addition problems except for the horizontal "equals" line (she knows the + sign from the √+ grades we have been giving). So we read the first problem together, top to bottom and left to right (the same rule as for reading a book):

> **". . . the very first thing we do when we see a math problem is to read it, just as we would read a sentence."**

Sample Transcript: Reading An Addition Problem

> *"One . . . plus . . . one ….. Good! So far, so good! Now, Sammy, this little line stands for 'equals' – can you say 'equals?' Good! Let's say it again, 'equals.' Good! What does the little line stand for? . . Good, that's right, 'equals.' 'Equals' just means 'is;' it means the same thing as saying 'is.' So let's start over and read the problem. One . . . . plus . . . . one . . . equals . . . two! Good! One plus one equals two! That is the same as saying 'one plus one is two,' because equals and is are just two different words for the same thing. They both mean the same thing. Let's read it again." [Repeat a few times.]*

The three key points here are (i) having the child read the math problem like a sentence, (ii) learning the important math word "equals," and (iii) learning that "equals" means "is." Although we will not mention this explicitly each time we discuss an addition problem, these three ideas should be emphasized in connection with every addition problem until the child has mastered them. Note that we will not generally provide the answer to the problem; we do it here just to facilitate the reading of the problem.

FIGURE 11.1: A graduate of the Plan uses the "red-block" technique to teach addition to her two dogs. (a) "One block," (b) "Another block," (c) "Put them together to add them," and (d) "Count them with your nose: One, two!"

o <u>*Concrete Approach to the Problem.*</u> In order to help the child get a concrete understanding of addition, we illustrate the problem by using small objects such as nickels, blocks, or fruit. It is best if the objects are identical, so that the idea of a numerical operation is not masked by the difficulty, for example, of adding apples and oranges! At the risk of being boring, let me give a detailed outline of one way to approach this concrete version of addition. Feel free to skip ahead if you prefer!

Suppose we are using red blocks. The exchange might go something like this:

Sample Transcript: Addition Using Blocks

"Sammy, that was very good reading of our NEW ADDITION PROBLEM! Well done! Now let's see what this means, what addition means. Do you want to do that? Or should we just go take a nap instead?" [If Sammy chooses the nap, that is a bad sign, especially at this exciting time! We have never had that happen, but if it does, you may want to shorten your homework sessions or give more emphasis to Field Work.] "OK, let's use these fun red blocks. Does it matter if we use red blocks or blue blocks? No, you are right, of course it doesn't matter. Let's use red blocks because I know red is your favorite color. OK?" [Notice how we drift back a bit from the new idea of addition and establish a relaxed tone by introducing the comfortable concept of a favorite color. Now we can surge ahead!]

> *"OK, so let's do 'one plus one equals two.' To do 'one plus one equals two,' you need to hold ONE block in your left hand – no, the other left hand – and ONE block in your right hand. Good! Do you see that we have one block here and one block here? Now, all addition or plus means is bringing them together, putting them together, and counting them. And you are very good at counting, aren't you? So bring the blocks together .. good! Now that they are together, go ahead and count them. Use your nose to count them if you want. One, two! Good!" [See Figure 11.1.] "So we get an answer – one plus is – how many was it? How many did you count? Two! Good! Let's do that again. Can I try it this time? OK, OK, you can do it again!" [At some point, try to take a turn so that you can demonstrate.] "Let's read the problem together, and you can use the blocks. Here we go . . . One – so where is one block, good – plus one – so we need the other block – good, now since we have a plus we have to bring them together, good, equals … count them … good! Equals two! So one plus one equals two – and that means the same as one plus one IS two."*

- <u>*Finger Approach to the Problem.*</u> Next, we take an approach to the problem that does not require any equipment or props (e.g., red blocks), but instead just uses the child's fingers. Under this method, two plus four is done by having the child hold up two fingers on her right hand and four fingers on her left hand, and then having her bring them together in front of her face, where she counts them – often by touching them sequentially to her nose (see Figure 11.2)!

FIGURE 11.2: Sammy demonstrates the use of the "finger method" to add "two plus four." Start with two fingers on one hand and four on the other, bring the hands together, then count the combined fingers (with your nose if necessary).

We accompany this with the same type of discussion that we used above in connection with the blocks, but emphasizing how wonderful it is that the child can do these addition problems even if there are not any blocks around. She will enjoy the feeling of adequacy and self-reliance.

- *Shape Diagram.* Try to get the child to point out that the circles are outside the boundary and the triangles are inside the boundary. For these more difficult counting tasks, one objective is to ensure that your child is using an appropriate strategy, e.g., row-by-row, top-to-bottom, left-to-right.

A Brief Diatribe About Learning Facts Versus Learning Skills. That was a very big Sample Homework, with many important new concepts. Take a breather for a minute while we discuss the difference between learning facts and learning skills. This diatribe is not merely an opportunity to give you a short breather; it will also help illuminate the theory of the Plan, and emphasize the stark contrast between the Plan's approach and "business as usual" in the American math classroom (the "Usual Approach"). Let me give you a bit of a heads-up, though, so that you do not begin to feel that we are adopting some sort of touchy-feely, New Wave philosophy. We will make three points below: (i) the Plan is a lot more fun than the Usual Approach; (ii) the plan teaches skills, the Usual Approach teaches facts, and skills are more important than facts; and (iii) facts are important too, but the Plan also does a better job of teaching facts than the Usual Approach. Please keep that third point in mind as you read through the first two; it will help reassure you that we are not Communists!

The standard approach to math education in the United States – the Usual Approach -- is to teach facts rather than skills; to require complete mastery of the rote facts from one level of math before moving carefully and gradually up to the next, incrementally higher level. For example, elementary school children learn their numbers carefully and in detail before they are introduced to addition; they master addition of one digit numbers by memorizing their early addition tables, then move glacially to the addition of simple two-digit numbers without carrying, then finally, weeks or months later, they begin to learn addition with carrying (known as "regrouping" to the corduroy-patch crowd). It is not uncommon for an teacher to provide "enrichment" to a child who has quickly mastered addition of single digit numbers by telling her to put a "1" in front of each number – i.e., 2+3 becomes 12+13 – as if this is a wrinkle that will take more than fifteen seconds to figure out, at least for a Pittsburgh Plan child! And more advanced skills such as multiplication and division are scarcely dreamt of, much less approached – they are forbidding specters from off the edge of the map, in "here there be monsters" territory!

Now, compare this standard approach with our work to date. We have introduced addition to a very young child at a time when she knows only four numbers! We will be introducing subtraction before she learns many more. And she will be doing addition with carrying before she is five, subtraction with borrowing (sorry, "regrouping") shortly thereafter.[2] What a difference! To begin with, our

2 And we will never do tessellations, because (i) they are of at most minor importance and (ii) they irritate our crack team of experts no end, because they are the archetypal example of confusing the learning of jargon with the learning of mathematics.

approach is a lot more fun than the standard approach of just memorizing things. Our children can actually solve problems! They have mastered new skills; they feel great about themselves. And practically every day brings a new challenge, a new adventure. This is not just a trivial benefit; it is large advantage, because it goes to the very core of a key Plan objective – helping children develop a life-long love of learning.

But there is more, much more. Which do you think is more important, developing a storehouse of memorized facts or learning to think better, to be smarter? Perhaps there was a time when this was a hard question, but certainly not today, in the age of information.

Let me give you an example. I do not know the capital of Angola. Possibly I should have learned it in school, but I did not – I was probably learning how to analyze an argument, or write a better sentence, or play guitar instead.[3] Anyway, suppose one of my friends, Barry Boring, did memorize that fact, presumably as part of a massive program of memorization of such things. How much of a real world advantage does this knowledge give him? What is his edge?

Well, as I write this, it is 11:39 PM on a Monday night. I went to Google's web page, entered "Angola capital city," and got back "Luanda" -- in <u>eleven seconds.</u> So, in a task that was particularly tailored to give him the maximum edge, Barry had at best an eleven-second advantage over the rest of the world. Pretty small return for a massive commitment of time and brainpower! Encyclopedic knowledge of precise facts is not useless by any means, but it has very limited value. This is, of course, bad news for all those children who devote great chunks of time to memorizing facts for spelling bees and geography bees. They and their parents are making a common mistake; they are confusing being knowledgeable with being smart. Someone who knows how to type fast and use a search engine will beat Barry Boring ninety-nine times out of a hundred in coming up with a specific fact. Barry's deep knowledge will win the day – by a little bit – in the rare case where the question happens to intersect his knowledge base, but in all other circumstances, the better search strategy will triumph. Skill trumps knowledge!

"Skill trumps knowledge!"

Or, think about it from another angle, the computer analogy that we mentioned briefly in an earlier chapter. A young child's mind is like a computer that is under construction. You can if you wish put your time and energy into building a huge memory/data base, and accept bargain basement microprocessors, interfaces and buses, etc. If you follow this path, your computer will be able to hold a great deal of data, but it will be slow to access it and process it, and it will be even slower in accessing information from the outside world. Or, you can go in the other direction and put your efforts into building a powerful CPU (central processing unit – like an Intel Pentium chip) and giving your computer high-speed buses, ports, and access lines. This computer can access information rapidly on a worldwide basis and then process it very quickly. Is it not obvious which is the better computer?

We are not done yet! Because, truth be told, facts are important; it is a good thing to have a strong

3 Or thinking about girls.

knowledge base. It may not be the number one objective of a rational approach to an education, but it is an important secondary objective. Here is a dirty little secret – the Pittsburgh Plan does a better job than the Usual Approach of teaching basic math facts. Memorization of facts by rote learning has two big flaws. First, it is crushingly dull, meaning that a child will have a hard time attending to the task at hand despite her best intentions. Second, the facts are not learned in a natural environment, and thus lack the rich context that allows the child to incorporate them into a vast network of associations. In a sense, the Usual Approach reflects a lack of understanding of what it means to "learn" a fact! [4]

The Pittsburgh Plan takes an alternative approach to teaching math facts, using a two-part strategy:

• ***Pittsburgh Plan Tactic #1: Don't Just Memorize the Fact; Use It!*** First, we present the child with many math problems of many different types. Because the child is presented with so many problems that she must attack and solve, the elementary facts become her allies, her tools. They are essential to her success. When our children figure out a fact, such as two plus three equals five, they are using it to solve a problem. Let me say that again; they are not simply reciting the fact; they are using it. This gives it a context, a sort of scaffolding upon which it can be hung in the child's brain.

Also, because the child is having fun doing math, and feels great about herself, she in enthusiastic and interested, a state that clearly promotes factual learning. Just think of the way that small boys can reel off sports statistics – batting averages, rebounds per game, etc. -- at the drop of a hat. These young boys certainly do not sit down and dutifully study such statistics; they learn them because they find the statistics to be interesting. This is one of the resources we are seeking to enlist for our math purposes.

• ***Pittsburgh Plan Tactic #2:The Advance Guard Strategy.*** Since we began this Chapter with a military analogy, we may as well discuss the Plan's second main tactic for teaching math facts in similar terms. The Usual Approach – the standard American school approach – to math complexity is to hunker down, circle the wagons, and creep forward in full strength at a careful, almost timid pace – sort of a pusillanimous General George B. McClellan approach, for readers who are familiar with their Civil War history. But what kind of a strategy is this? McClellan squandered many an opportunity because of this strategy, and so does the Usual Approach. The great Prussian military strategist Carl von Clausewitz knew better, writing almost two hundred years ago:

> "Every body of troops, when not completely in readiness for battle, requires an advanced guard to learn the approach of the enemy, and to gain further

4 That said, some rote memorization is a necessity. If the schools did not do it, we would; we just would not beat it into the ground, or elevate it above other types of material.

> particulars respecting his force before he comes in sight, for the range of vision, as a rule, does not go much beyond the range of firearms. But what sort of man would he be who could not see farther than his arms can reach!"[5]

Certainly, our experience with the Plan has made it very clear that our children want to see as far as they can – they are not afraid of math complexity; they relish it. Or, how about one more military analogy? (We promise this is it for the entire book!) In early 1942, the United States was reeling as Japan followed its devastating bombing of Pearl Harbor with a rapid series of successful attacks on Hong Kong, Malaysia, the Dutch East Indies, Wake Island and Guam. Japan seemed invincible, safe from attack as its powerful navy won battle after battle. American morale was at an all-time low; there were even fears that Japan might soon strike the American mainland. At a time when the natural inclination would be to step back, erect strong defenses, and protect what was left, America instead struck back. On the morning of April 18, 1942, a small group of American B-25 bombers broke through Japanese defenses and struck the very heart of the enemy, Tokyo itself, in a daring daylight raid. Although the attack was not militarily significant, it stunned Japan, energized the America populace, and changed the course of the war.

What is the point of all of this military analogizing? It is simple: we Pittsburgh Planner are not sissies, and neither are our children. We are not afraid to venture into the vast and complex world of mathematics; indeed we charge freely ahead, establishing advance guards and outposts far in advance of our current position. For example, once we have learned our first four numbers, we do not simply consolidate and learn another twenty or thirty numbers. We strike immediately deep into enemy territory; we use those four numbers to establish an initial outpost in "addition land" and "subtraction land." And we will continue this aggressive pattern of moving quickly to claim vast amounts of mathematical territory. Of course, we consolidate also, but we do so almost as an afterthought in the context of our race to complexity.

" . . . once we have learned our first four numbers . . ."

" . . .we use those four numbers to establish an initial outpost in "addition land" and "subtraction land." "

Why does this approach work so well? For three reasons, at least: (i) it is much more interesting than the tedious General George B. McClellan consolidation approach – the Usual Approach of our standard American program of math education -- so our students are engaged and attentive; (ii) the rapid acquisition of mathematical skills provides many more applications of the basic facts of earlier levels, making those facts more useful and thus easier to memorize; and (iii) by establishing

5 Clausewitz's On War, Chapter VII (trans. 1873 by J.J. Graham).

early outposts in unfamiliar terrain (e.g., very simple addition problems), we give our children a solid base from which to explore further – i.e., we take away the fear that accompanies the Usual Approach's treatment of complex mathematical terrain as that "undiscover'd country from whose bourn no traveler returns." You will see that the Field Work for this Chapter includes a song covering a few advanced addition facts; this song is another example of establishing an outpost in the land of advanced addition.

3. *Sample Homework #35.* We give the child a short break from the exciting new skill of addition. (Remember the Standard Discussion, about how we introduce things many times and do not expect the child to get it the very first time? Well, here we see that approach in action.) Now, at least for a moment, we turn our attention to reading, specifically, the word "cat," although we do not tell the child what that word is just yet. It is her job to try to figure that information out for herself!

- *Sounding Out Words.* When we are teaching the child to sound out words, we approach the task in steps.

  - *Letter By Letter.* First, we go through the letters individually, possibly several times, having the child make each sound, making sure that the sounds of the letters "c," "a" and "t" are firmly in the child's immediate memory.

  - *In a Row Without Pausing.* Then we ask the child to make the sounds right in a row, without stopping. She will do so, probably in a very staccato fashion – almost certainly, she will not make the leap to link those sounds into the word "cat." That is OK, it is completely normal. We have had children take many months to go from making the letter sounds in a staccato way to linking them into a word – when the light finally goes on, it is a wonderful thing!

  - *"Slide" the Letter Sounds Together.* In order to make some inroads into this difficult mental barrier to reading, we encourage the child to try to "slide" the letter sounds together, so that they flow one into the other. Show this by example with "c" and "a," and then with "a" and "t," trying not to let her guess the word from your pronunciation.

- *Circle Entire Word.* Because we are now thinking of the three letters "c," "a" and "t" not as individual letters, but as one word, we ask the child at the end of our work on the word "cat" to circle the word as a whole. This reinforces the idea that it is a single unit (which will be important when we begin to lay the groundwork for reading by word recognition as well as by phonetic "sounding-out").

4. ***Sample Homework #36.*** We introduce the letter "s" and the number "5," and continue with our early approach to the two dominant themes of this Phase, reading the "at" words and addition/subtraction skills.

5. ***Sample Homework #37.*** We introduce "H," "v," "y" and "6," and begin a brief, 3-Sample-Homework-long hiatus from addition and reading "at" words. Again, we are following our general plan of multiple introductions to difficult new material, with a low-key attitude that is designed to defuse pressure and tension.

6. ***Sample Homework #38.*** We introduce "p" and the number "11" (just for fun – a small, easily recognized outpost in the land of two digit numbers). The shapes diagram should be used to reinforce counting strategies (left to right, top to bottom) and also the different shapes ("Count the triangles, now count the rectangles," etc.). See Figure 11.3 for an actual homework done by Sammy based on Sample Homework #38 – note that she insisted on copying the number "11"!

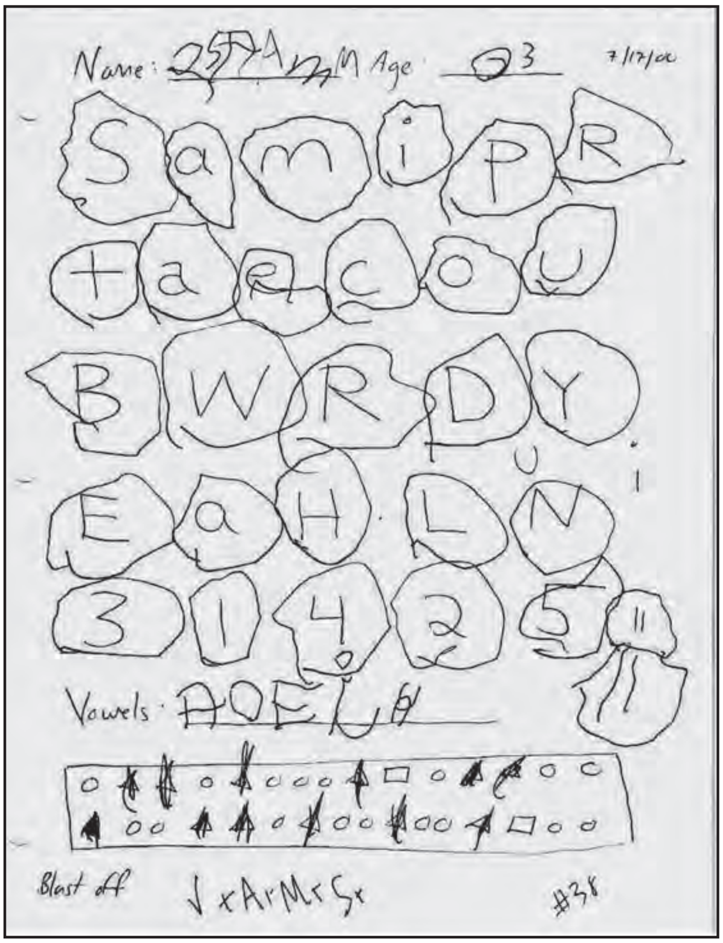


FIGURE 11.3: An actual homework done by Sammy based on Sample Homework #38. Note that Sammy tried to copy the exciting new number (11), circled each item after she read it, wrote the vowels herself (out of order), and marked the triangles to ensure one-to-one correspondence as she counted them.

7. *Sample Homework #39.* We introduce two more letters that help with the "at" cluster of words, "b" and "r," and also the number "8." Note that we are using our period of "stepping back" from addition and reading to consolidate and expand our lower level knowledge of letters and numbers.

8. *Sample Homework #40.* "G," "x" and "7" are introduced. Also, we are back to our two main themes, "at" words (now including "fat") and addition.

9. *Sample Homework #41*. We introduce "X" as well as "h" and "F" – two more "at" word letters. We add the word "hat" to our reading list (and also "SAT" at the end of the first row of letters – we will use "Sat" as our "at" word in the next Sample Homework). At some point soon -- now would be a fine time, in fact – you should focus on the fact that "cat," "fat," "hat" and "sat" all end in "at" and all sound alike – they rhyme. Work with your child on making the initial letter sound and then adding the sound for "at." The goal is to think of the "at" letters as a single unit having a single sound. You could, for example, have your child circle the "at" as one unit in each "at" word that she encounters.

"The goal is to think of the "at" letters as a single unit having a single sound."

10. *Sample Homeworks #43-47*. You probably would not notice this unless we told you, but these Sample Homeworks are duplicates of Sample Homeworks 38-42. Again, we using our approach of forging ahead, then pulling back; of daring advance followed by strategic consolidation. And, we share this approach with the child; we make her our partner in this endeavor.

- *Horizontal Addition.* Note that there are three important new ideas flowing from an apparently minor change in Sample Homework #47, namely, the switch from the vertical (top-to-bottom) addition used in Sample Homework #42 to horizontal (left-to-right) addition.
    - o *Equals Sign*. First, at the most concrete level, we introduce the equals sign, "=".
    - o *Reading Addition Horizontally.* Second, also at the concrete level, we help the child learn a new way of writing an addition problem (horizontally). We make sure that she can read this form of addition as well as she can read the earlier top-to-bottom form of addition.
    - o *Two Different Names For the Same Thing*. Third, at a more abstract level, we point out that these two different forms of addition are just two different ways of writing the same thing, like "sam" and "SAM."

11. *Sample Homework #48.* Here we introduce the letter "g" and present the child with a very difficult shape counting exercise. More importantly, the hiatus is over; we are now getting serious about addition.

- *Three-Part Approach*. Remember to do three things with each addition problem:
    - o have the child read it like a sentence;

- o do the addition using concrete objects such as red blocks (you should continue to use this concrete approach for addition problems periodically throughout this Phase, fading out thereafter); and
- o use the finger method.

• *Encourage Memorization!* But, there is one proviso to this three-part procedure! After your child reads the problem, ask her if she remembers the answer – if so, she (or you) can just write the answer down and move on! In other words, there is a real-life benefit to your child from remembering these answers – she saves a lot of time by not having to plow through the two methods of solving the problem.

• *At This Point, the Child Should Be Doing a Lot of Writing In the Homework Sessions.* This provides us with a good segue into one final idea: at this time, your child should routinely be writing her name (and possibly the date and even her age, if you want to add that) at the top of the page, copying a few letters and numbers, writing answers to math problems, etc.

12. ***Sample Homework #49.*** We welcome the number "9" to our stable, and resume "at" word reading.

13. Sample Homework #50. The big news here is that we introduce zero, the most important of all the integers (although the number one probably runs a reasonably close second).

• *Zero As a Placeholder.* Why is zero so important? Well, the big reason, as you probably know, is its use as a placeholder, allowing us to use the place system of numbering (i.e., using the ones column, tens column, hundreds column, etc.) instead of being stuck with a system such as Roman numerals. We cannot really make that point quite yet to the child, because we have not done much with two digit numbers. So we simply whet her appetite; we tell her that zero is the most important number for reasons that she will soon learn. [6]

• *Zero As the Additive Identity*. Zero is also very important because it is the "additive identity" – i.e., it is the only number that you can add to another number without changing the other number. For example, if you start with six and add zero to it (6+0), you still have six; nothing has changed. We will tackle this is the next Sample Homework; for now, just be a little mysterious about it.

• *Zero As "None."* Go ahead and teach your child that zero means "none" - ask her to hold up zero fingers,

---

6 Actually, although zero is the most important <u>integer</u>, it is probably not the most important <u>number</u>. If pressed – or even if not pressed! -- I would rank the five top numbers in order of importance as follows: (i) *e* (among many other things, the base for the natural log, known as " ln" and pronounced "lon" as in Lon Chaney); (ii) π (pi, the ratio of the circumference of a circle to its diameter, at least in flat "Euclidean" space); (iii) zero; (iv) i (the square root of negative 1); and (v) 1 (the multiplicative identity). Amazingly, these five numbers are all contained in one version of Euler's famous equation: $e^{i\pi} + 1 = 0$.

ask her how many giraffes she has as pets, etc.

• *Zero, the Letter "O," and a Circle*. There is one other thing that you can do with zero in this homework session. Focus on the fact that we now know three meanings of the same symbol: a shape (circle); the letter "oh;" and the number zero.

• *Confusing Numbers: 2; 6; and 9*. Like many children, Sammy had trouble distinguishing the numbers "6," "2," and "9." We tackle this in the last line of numbers using one of our favorite techniques, repetition, and also emphasizing both recognition and production (filling in the blanks with 6's). See if your child spots the pattern and figures out that 6's should be used to fill in the blanks; if not, give her a little guidance.

14. Sample Homework #51. We introduce "Q" and then move on to more important things.

• *Reading Outposts*. First, we play around a little bit, establishing a few early outposts in "reading territory." Notice the word "cAT" buried in the second line, and notice that the fourth line is the word "Hi." Finally, in our list of words we have the usual ("Cat") but also a new fun word, "Dog." Just go ahead and sound these words out with your child, emphasizing that it is not part of the regular program; it is just for fun. Focus particularly on the new vowel sounds in these words.

• *Sixes and Twos*. Again, we emphasize sixes and twos.

• *Adding Zero*. Now, we have an addition problem involving zero. Go through this problem in the usual three ways (see the discussion of Sample Homework #48 above), and then discuss the fact that the answer is the same as the original number – that adding zero did not change anything. Talk about this for a while. Mention, after a while, that this is one reason why zero is so special. Then, ask your child why zero is so special. Go through this a couple of times, seeing if she can produce an explanation of the importance of zero. Ideally, she will soon (over the course of a few homework sessions) be able to extract a useful rule from this idea; namely, that if one adds zero to a number, the number just stays the same, so she can just write down the answer and move on.

> **"useful rule . . . one adds zero to a number, the number just stays the same . . ."**

15. Sample Homework #52. Two big things happen here:

• *The Number 10*. First, right after the Sample Homework introducing the importance of zero as a place holder – Quelle Coincidence! – we introduce the number "10." Here, discuss how useful zero is in writing this number. Without the zero, it would just be "one;" because we have zero, we do not need to invent a new symbol (draw a smiley face or some such thing) for this new number, we can just put two of our "old friends" – "1" and "0" -- together to make the new number.

- *Subtraction!* Second, we introduce subtraction! Go ahead and have the Standard Discussion about introducing new ideas and not expecting the child to understand them right away. Then, approach the subtraction problem from several different angles:

    - o <u>*Read the Problem*.</u> First, as usual, read the problem. Take a few minutes to help your child learn the "minus sign." Teach that it means "minus" or "take away."

    - o <u>*Concrete Approach (Red Blocks)*</u>. Second, tackle the problem using the concrete approach – maybe red blocks again. Emphasize that subtraction is the opposite of addition – that we will be taking things away rather than putting them together or adding them together.

    - o <u>*Fingers*.</u> Third, use fingers to do the problem, "taking away" or "minusing" by folding fingers down. Discuss the feasibility of the alternative approach of chopping off fingers to take them away (all in all, not a good idea). Maybe even discuss what would happen if you subtracted zero! (We will get to that eventually in the Sample Homeworks.)

- *"At" Words.* Finally, we read another "at" word, "SAT."

16. ***Sample Homework #58.*** "Z" and "f" – a useful "at" word letter – join our repertoire.

17. ***Sample Homework #60.*** Sammy begged for a homework session on one her days off, so we did this short one:

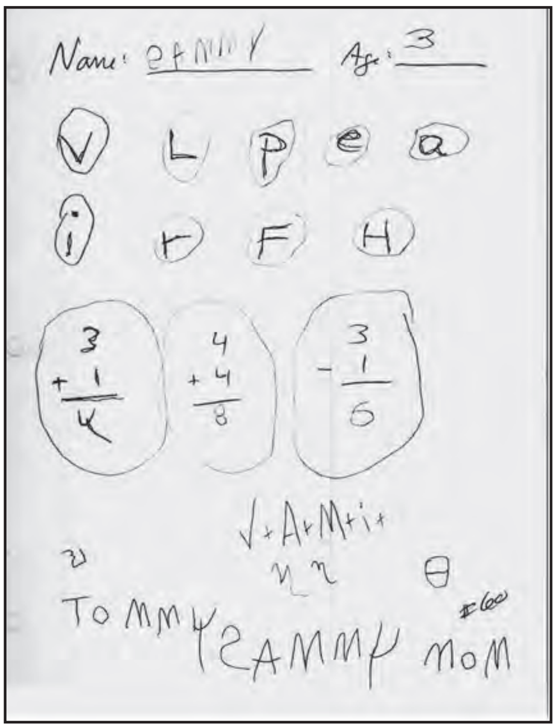


FIGURE 11.4: **Sammy's work on Sample Homework #60. Note the backward "S" in Sammy's name and the difficulty in writing "2" in the subtraction problem. Typically, we do not worry about these types of details when a major skill is being learned; instead, we correct these details during a period of consolidation. Note also Sammy's extra work - writing some names at the bottom, and asking for and copying a few Greek letters.**

18. ***Sample Homework #61.*** Now, for fun, we add another component to the grade that your child receives after every homework. Recall, your child has been getting a "check-plus," a letter grade of her choice (A+, X+, etc.), "VE___" for "Very Excellent _____" (e.g., "VEAP" for Very Excellent Addition Problems" – see the discussion of Sample Homework #15 above). You may enjoy asking your child to help fill in the blanks in the "VE__" grade – but that is not the new grade component. What we are adding now are Greek letter grades: α, β, γ, δ, ε, ζ, η, θ, etc. Some of the more interesting Greek letters are:

α alpha	β beta	γ, Γ gamma	δ, Δ delta	ε epsilon	ζ zeta	η eta	θ , Θ theta
ι iota	κ kappa	λ, Λ lambda	μ mu	ν nu	ξ, Ξ xi	ο omicron	π pi
ρ rho	σ, Σ sigma	τ tau	υ upsilon	φ, Φ phi	χ chi	ψ, Ψ psi	ω, Ω omega

FIGURE 11.5: Lower case and selected upper case Greek Letters.

Start with one Greek letter, and ask your child to reproduce it. Eventually, you can work your way up to two or three Greek letters, and have your child select and write down the same number of her own Greek letters. Children love to do things like writing "Dad η π" – "Dad *eta pi*." (Get it?) They are also especially intrigued by the Greek letter ψ ("psi", known to children everywhere as "spy"). The point here is to teach your child something special – advanced knowledge that will make her feel able and proud. Children love to know letters that no one else knows!

19. ***Sample Homework #62.***

- Notice the vowels in order in the first row of letters.

- Also note that we have extended our "at" cluster work to a related word – a very familiar one – "Sam." Here, focus on the word "Sat" and how "Sam" is similar and how it is different. If possible, try to get your child to view the first two letters "Sa" as a unit that makes a single sound, and then add the different ending sounds ("t" and "m"). As always, encourage and reward word recognition, but in this case even if your child recognizes the new word, go through the above discussion.

- And, note the subtraction problem involving the subtraction of zero – go ahead and discuss this, comparing the effect of subtracting zero to the similar effect of adding zero (i.e., no change; zero is the identity for both addition and subtraction).

- Soon we will introduce the letter "d," and most children will have trouble distinguishing "d" and "b" at that point. As a preemptive maneuver (this is not a military analogy, it is a bridge analogy),[7] you might want to try having your child draw an extra, top loop on the small "b" to turn it into a capital "B." This may turn out to be a helpful mnemonic device for her.

20. ***Sample Homework #62A*** (see Figure 11.6). This homework sheet is not technically one of our Sample Homeworks. It is one example of the many times when Sammy wrote her own homework sheet on an off day. Needless to say, this is a very good sign!

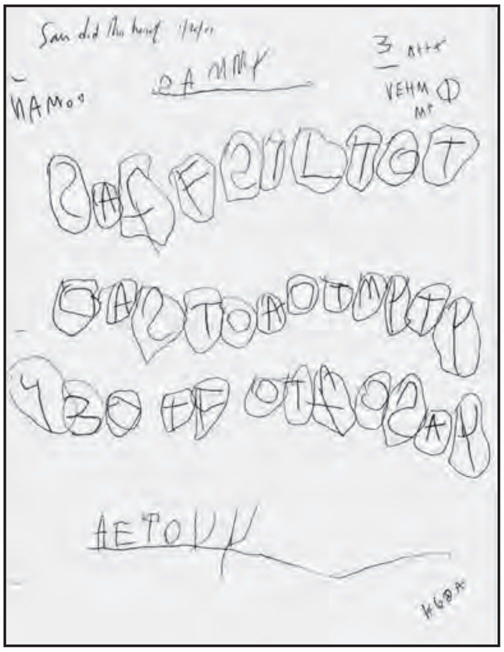


FIGURE 11.6: **Sammy made and did this homework sheet herself, on her own. Do the Junior Kumon kids do this?**

21. ***Sample Homework #63***. Ground control to Major Sam: "you are ready for liftoff; next we start doing BIG GIRL work!" This is the final Sample Homework of Phase 3, and it hits a little bit of everything. We even introduce a script *"a"* just for fun! Note that four "at" words are listed as a sort of graduation exercise; if your child can read them, she has graduated from pre-reading and is ready to start Phase 4, where we do some serious reading work. Congratulations! Have a Phase 3 graduation ice cream with your child.

"Congratulations! Have a Phase 3 graduation ice cream with your child."

PART 2: Field Work For Phase 3

Field Work continues to be very important throughout Phase 3, and should include the following:

- Continuation of the Phase 2 Field Work from Chapter 10;

7 The card game kind of bridge, not the River Kwai kind of bridge.

• ***Special Flash Card Notebook.*** Buy a small spiral notebook where the pages are index cards, preferably 4" by 7" or larger. (You can find this kind of notebook in most office supply stores and stores selling school supplies.) Cut the cards all the way through, from the outer edge to the spiral wire, so that each of the three pieces is still attached to the spiral wire. The result is that each card is divided into thirds, with each third being able to turn separately. Then write the capital letters from "A" to "Z" on the left-most pieces, one letter per page. In other words, write the letter "A" on the left piece of the first card, the number "B" on the left piece of the second card (directly underneath the card piece with "A" on it, etc.). Then write the lower-case letters "a" through "z" on the first twenty-six middle pieces and on the first twenty-six right-hand pieces. Since each third of a card can turn separately, you have the ability to make any three-letter word by simply turning the cards appropriately. For example, the word "Ate" can be spelled by keeping the first card on the left side ("A"), flipping to the twentieth card in the middle ("t"), and flipping to the fifth card on the right side ("e"):

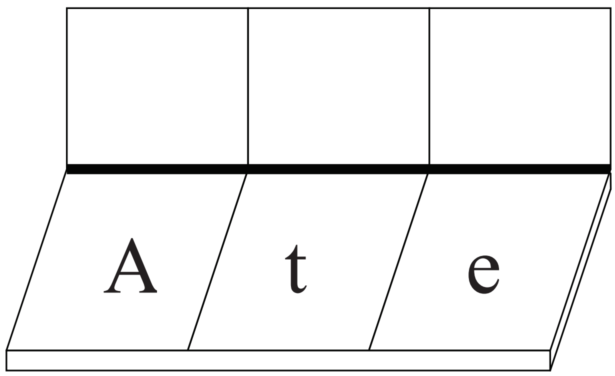


FIGURE 11.7: An index card notebook, with each card cut into three pieces that can be flipped up or down independently. The capital letters are written in alphabetical order on the left pieces; each of the other sets of pieces has the lower case letters written in alphabetical order. Two and three letter words can be formed by flipping the cards appropriately.

Use this book to make "at" words and other three-letter words that your child can sound out. Your child will value and enjoy this amateur construction much more than a professional analogue because you made it – it shows her that you are truly a part of her learning process;

• Read to and with your child frequently;

• Play rhyming games, especially ones involving "at" words, but also using other words as well. For example, say "sat" and ask your child to produce a rhyming word, then switch, letting your child give you a word;

• Now try a variant on the rhyming games. Give you child a word -- for example, "sat" – and ask her to give you a word that has the same sounds except that it ends with a different letter (e.g., "sat" and "sad"). Or just focus on the vowel sound – ask your child to give you a word that has the same vowel sound as in the word "hit," for example. Again, switch roles from time to time;

- Practice recognizing words without sounding them out. For example, we often teach the child the word *"the"* at about this point, noting carefully that it is a very hard word to sound out – "so hard that we will not even try sound it out yet. Instead, we will memorize it, because it is very common, we will see it all the time, and it is nice to know what it is." Then, as you read a book with your child, point to the word *"the"* from time to time and ask your child to identify it. (By the way, we italicized the word *"the"* in this paragraph because it is so common the eye skips over it unless drastic steps are taken to call attention to it!);

- Give your child lots of paper and many opportunities to write or draw;

- Reinforce the Standard Discussion (about approaching new things many times, and not expecting the child to understand them the first few times) in conversations involving the child and, for example, other family members – e.g., explaining that approach to the other family members;

- Practice addition and subtraction with concrete objects (blocks, coins, raisins, etc.);

- Practice doing addition and subtraction problems using fingers in the car, or while waiting in line at the ice cream stand, etc.;

- Sing the "I Don't Know Any More" song (see Figure 11.8 below); and

- Give the child sounds – such as "s" and "ick" – and ask her to put them together into a word.

Phase 3 is ended, we have crossed the Rubicon. Now it is time to storm Rome itself!

FIGURE 11.8: **A song for the ages!**

PART 4:
The End of the Beginning . . . .

Chapters 12 and 13 describe the final two Phases of the Pittsburgh Plan. By the end of Phase 5, you will find that your child has become a reader and an able mathematician. Congratulations!

Chapter 12: *Do You Believe in Magic?*

In late 1926, as he lay dying from the effects of an infected appendix (exacerbated by punches to the stomach which he had invited as a demonstration of his astonishing strength and fitness), the greatest magician who ever lived dictated a brief note to his personal secretary, a note that soon would serve as his de facto epitaph. In it, this man who was noted for his unparalleled skill, grace and athleticism – who was in fact suspected by many of possessing true magical abilities and mental powers -- explained the source of his magic:

> "What little success I may have had has come from making up my mind in early youth to be the best in my line no matter what it cost in hard work and never to deviate from that course.
>
> People often commend me for my courage; often say, indeed, that I am a performer of miracles of courage. This is far from true. I do nothing anyone else could not do with equal practice and years of toil.
>
> When I train to jump from a high bridge, don't think that I jump from the great height all at once. I [do] it all by gradual stages. I get a ladder and each day jump from one rung higher than the day before. When I scale a seven story building in a certain motion picture, I started by climbing up one story and coming down, then two and coming down, etc., till I had gone the seven. This is the whole secret of getting to the top in anything."[1]

Magical results, one step at a time. It fits! And that is why we call Phase 4 the "Houdini Phase."

PART 1: Detailed Discussion of Phase 4: The Houdini Phase

***Sample Homeworks*:** Sample Homeworks ##64 – 114.

Summary. Because we cover so much ground in this Phase, most parents appreciate having an up-front understanding of the game plan. Of course, our children do not need such an overview; they are perfectly happy just sailing along performing their usual inductive magic on mountains of challenging new material! But we adults are made of less stern stuff; we feel more comfortable with a road map. So, for the benefit of the feeble oldsters out there (i.e., anyone over the age of fourteen), here is our Phase 4 strategy in a nutshell:

General Approach to Both Math and Reading. Our general approach is to charge forward at breakneck speed into exciting new areas in both math and reading. We establish strong outposts in advanced territory, and then use these outposts as foundations for consolidating our advanced positions. Like Alexander the Great and Genghis Khan,[2] like Microsoft and Intel, we

1 Ehrich Weiss (aka Harry Houdini), "Houdini Laid Success to Hard Work," New York American, Nov. 2, 1926.

2 We are not the only ones to use a "conqueror" analogy. Karl Gauss, perhaps the greatest mathematician of them all, said, "It is

grab territory, then consolidate.[3]

Specific Approach to Reading. We want your child to be reading sentences on her own – reading simple books on her own! – in seven months. Many knowledgeable people will criticize this objective as being ridiculous; for example, we have been told by several elementary school teachers that children simply cannot read sentences on their own until they are five or six years old. They say it cannot be done; we do not have much time to deal with such criticism, because we are too busy doing it! Here is how we establish a beachhead in reading territory:

- We work hard on phonetic reading, emphasizing the process of sliding letter sounds together to make words;
- We learn two key rules that help us read words with multiple vowels, the Two Vowel Rule and the Silent "e" Rule;
- We memorize a number of common words, and encourage reading by word recognition;
- We encourage the use of contextual clues in reading; and
- We learn about important aspects of sentence syntax.

Specific Approach to Math. Meanwhile, we strike like lightning on three different math fronts:

- *Addition and Subtraction.* We learn the important "call yourself" methods of addition and subtraction, and also establish outposts of known addition and subtraction facts by learning the "Zero Rules."
- *Fractions.* We learn to read fractions carefully, to understand a fraction's syntax, and to understand the geometric meaning of a fraction.
- *Intersections.* Through complicated intersection drawings, we learn a great deal about the abstract ideas of boundaries, interiors, etc.

Phase 4 Objectives. Here is the customary complete list of important objectives for this Phase:

not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment. When I have clarified and exhausted a subject, then I turn away from it, in order to go into darkness again . . . . I imagine the world conqueror must feel thus, who, after one kingdom is scarcely conquered, stretches out his arms for others." K.F. Gauss, Letter to Bolyai, 1808.

3 Of course, for the benefit of all those Microsoft and Intel attorneys reading this book, let us emphasize that nothing in this sentence is intended to suggest any parallel between these admirable, consumer-friendly corporations and any legendary rapacious conquerors!

- Continue to reinforce the letters, shapes, numbers and vowels (and especially vowel sounds) from earlier Phases;
- Continue to do Field Work regularly;
- Introduce all two-digit numbers and all remaining letters;
- **Introduce the "call-yourself" method of addition;**
- **Introduce the "call-yourself" method of subtraction;**
- **Read using three methods: phonetics; recognition; and context;**
- Memorize key words: the; I; to; and; do; me; he; go; no; you; and your;
- Introduce sentence structure: periods; question marks; exclamation points; commas; and quotation marks;
- **Teach the Two Vowel Rule and the Silent "e" Rule, also known as the Boss "e" Rule;**
- Teach techniques for reading two consecutive consonants;
- Emphasize the most important rule of all for reading English: Every rule has exceptions, including this rule;[4]
- **Introduce fractions: vocabulary; syntax; and geometric meaning;**
- Memorize the Zero Rules (adding and subtracting zero; a number minus itself equals zero, etc.);
- Learn other reading techniques: "s" for plurals; "th" sounds; "ing" words; "ay" and "oy" sounds; etc.;
- Introduce two-digit addition;
- Introduce intersection pictures to develop a deeper understanding of inclusion and boundary concepts; and
- Work on remembering and understanding written sentences (as opposed to just pronouncing them correctly).

Time Frame for Phase 4. Sammy took approximately seven months to finish Phase 4, starting at age 47 months and finishing at age 54 months. Again, homeworks were often done multiple times, with the result that there were approximately eighty-five homework sessions over the seven months of Phase 4. This works out to almost three sessions per week, short of the recommended dosage but at least in the ballpark. Meanwhile, Field Work remained important, but there was a large added element of outside activity: near the end of this Phase Sammy began reading voraciously on her own. She was thrilled at her ability to tackle the printed word without help, and spent hours each day puzzling through simple story books.

4 "What," you ask, "is the exception to this rule?" The answer is simple; this rule is the exception to itself; it has no exceptions. Think about that one for a minute! We have ventured into Gödel, Escher and Bach self-referential territory. Douglas R. Hofstadter, Gödel, Escher, Bach: an Eternal Golden Braid, Basic Books, 1979 (Pulitzer Prize Winner, 1980).

Detailed Suggestions and Comments for Phase 4:

1. A General Comment: Do Not Be Alarmed By the Rapid Pace. In this Phase, we move quickly and cover a great deal of ground. Do not be alarmed! We may be sprinting, but we are sprinting carefully.

- *Outposts First, Consolidation Second*. It is a core part of the Plan's approach to introduce new concepts well before prior concepts are mastered in their entirety. In particular, we do not await the complete memorization of facts such as number identification before introducing new skills such as addition. Memorization is a separate mental task from mastering procedures and skills; there is no reason that they cannot occur in parallel. Recall that our theory of math learning relies heavily on enlisting the child's language learning skills as our ally, since math is, after all, just a form of language. From this perspective, the usual practice of requiring complete memorization of relevant math facts at each level before moving on to the next level (e.g., number identification before addition, addition facts before multiplication, etc.) is akin to requiring a child to memorize the entire lexicon of words before using them in sentences! This is not how a child learns; this is not how we keep a child interested; in short, this is no way to run a railroad, or a learning program for a child!

- *A Deliberate Sprint.* Our attention to cognitive challenge and progress has another corollary as well: we want our children to feel great about themselves as they stretch and challenge their intellects. This means that (i) the emphasis of each homework – at early ages, a large emphasis – should be a review of things that have been mastered recently (the Sample Homeworks have been structured carefully to satisfy this objective), and (ii) it is good to repeat homeworks. These cautious steps – these leisurely moments during our headlong cognitive sprint -- have three benefits. First, they ensure that concepts are truly understood and internalized. Second, they allow the child to enjoy a sense of mastery, and to proceed without pressure to new things that are introduced in a casual and relaxed manner. Third . . . well, the third benefit springs from the fact that our program is a vast and radical departure from the normal curriculum presented to young children; we are rocketing along at a clip that is many times faster than the norm! We do not want the child ever to feel that she is on a roller coaster, so we cover these vast distances one small stride at a time; our weapons are in fact not those of a sprinter, but those of a marathoner: persistence; consistency; and patience. We are not Krakatoa; we are the desert wind scouring the Mogollon Rim of Arizona. In short, because our overall program is so aggressive and fast, we must be cautious and patient in its implementation.

- *A Child Shall Lead the Way.* Perhaps we should discuss this third benefit – staying within the child's comfort zone – a bit further. The whole idea of the Pittsburgh Plan is that we are not pushing the child; we are simply ceasing to act as impediments. We are presenting the child with opportunities and allowing her, and her natural curiosity and eagerness, to set the pace. Thus, as we introduce new things, we monitor the child's reaction. Is she excited and happy? That is the ticket; that is the goal. Does she seem a bit overwhelmed? Pull back immediately to more familiar ground. The new material can wait; there is always another day.

A Motivational Math Moment.

- "No one really understood music unless he was a scientist, her father had declared, and not just a scientist, either, oh, no, only the real ones, the theoreticians, whose language mathematics. She had not understood mathematics until he had explained to her that it was the symbolic language of relationships. 'And relationships,' he had told her, 'contained the essential meaning of life.'" Pearl S. Buck, The Goddess Abides, Pt. I, 1972.

- "[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word."
Galileo Galilei, Opere Il Saggiatore p. 171

- "I remember once going to see him [Ramanujan] when he was lying ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. 'No,' he replied, 'it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways.'"
Godfrey H. Harding, Ramanujan, London: Cambridge University Press, 1940.

2. Sample Homework #64. "J" and "K" are introduced; note the pattern of letters in the first line and see if your child spots it. (Help her if she does not.) More importantly, this Sample Homework introduces your child to her first sentence! This is just a brief exploration, an outpost, if you will; we will back off of it in the next few homework sessions. But, meanwhile, with appropriate fanfare, you should cover the following in this session:

- *The Idea of a Sentence.* Discuss the idea of a sentence; i.e., a group of words that fit together to say something. Ask if the group of words "fat cat hat sat" make a sentence. Use examples to convey the idea that a sentence is a group of words saying something.

- *Read the Sentence.* Have your child read the words, and then repeat them. Discuss the word "the" for a while – she should have been working on it in Field Work.[5] Go over the sentence a few times. Discuss what it means, and that it tells a short little story. Discuss the story; perhaps your child can use her imagination to elaborate on it a bit.

- *Punctuation (the Period).* Point out that it is easy to spot sentences because of "punctuation" – small symbols that send us signals about how words are connected together. Here, we have the most important (and easiest) punctuation, a period. Point out that periods mark the end of the sentence, the end of the idea.

- *Capitalized First Letter.* Mention that the first letter of the first word in a sentence is capitalized – yet another clue that we are looking at a sentence.

3. Sample Homework #65. Here we back off of the idea of a sentence and just provide a list of words (featuring a new sound, "op"). Ask your child if these words are a sentence, and discuss why they are not. We introduce "j," "l" and "n."

4. Sample Homework #66. We misspelled Sammy's name in the first row of letters, omitting a letter. She enjoyed filling in the missing letter. On the math front:

- *Sums Greater Than Ten*. There are two addition problems requiring more than ten fingers – see how your child handles them. We are motivating the "call-yourself" technique of addition, which will be introduced in Sample Homework #68.

- *Reminders.* A couple of reminders about math problems:

    - *Read the problem*. Always have the child read the entire problem aloud, emphasizing the operation, e.g., "Four <u>minus</u> one equals . . . ." This will help her focus on the task – addition or subtraction, etc. Also, it will prepare her for the important idea that math symbols can be replaced by words, and vice-versa, which will help greatly with word problems and algebra (for example, "of" means "times;" one-half "of" sixty means one-half "times" sixty).

    - *Use Proper Vocabulary.* Do not use baby talk – do not eschew the use of words such as "subtraction" and "addition" in favor of "take away" and "plus." Your child is a veritable language learning machine, and will have no trouble at all with correct terminology so long as you pronounce it clearly, use it consistently, and, most importantly, do not display a fear of it. Your child will sense any doubts or fears you have, including worries that a word or concept is too hard for her.

5 See Chapter 11, Part 2.

5. Sample Homework #67. We introduce "k." More importantly, we push forward in the areas of reading and fractions:

- *Reading – Sliding Letter Sounds Together to Make Words.*

  o *Remain Calm!* One of the hardest steps in becoming a reader is learning to hook letter sounds together into words. You may well find it frustrating; your child will seem to be poised to put sounds together into words, and may even be reading the familiar "at" words, and yet she may not make that final reading leap for many weeks or months. Do not panic, and do not push your child. This is entirely normal; she will make the connection when she is ready to do so. Pushing will merely infect your child with your own frustration! Simply continue, calmly, to provide opportunities so that she can make the leap when she is ready. Remember that our approach is not to push the child beyond her natural limits, but instead to remove artificial barriers to her progress. Conventional educational theory greatly underestimates the child's ability; we are content to let the child drive the pace of progress.

  > **"We are content to let the child drive the pace of progress."**

  o *Preliminary Sound Sliding: Syllables.* In order to help your child learn to "slide" letter sounds together so that they merge into word sounds, we try to help her think in terms of syllables rather than letters. If we can get her to think of "cat" as being a combination of "ca" and "t" sounds (or "c" and "at" sounds), then we are halfway home; that is a significant step beyond thinking of all three letters as separate sounds. The words in this Sample Homework are designed to encourage this kind of syllabic thinking.

  o *Word Groups Having the Same Ending Sounds (Rhyming Words)*. First, we focus on groups of words that have the same last two letters but differing first letters (i.e., groups of rhyming words). There are four such groups of words in this Sample Homework: (i) the "at" words (hat, sat, rat, fat, mat and tat), which are located in the top row of words; and (ii) three pairs of rhyming words which occur in the following row: ham and sam; ran and fan; and map and tap. Take these clusters of rhyming words, one group at a time (you might want to draw a light "box" around each group), and discuss them with your child. Have her read them to you, then read them back to her. Ask her to listen carefully while she reads and also while you read. See if she can hear how the words sound the same; discuss that they sound alike because they end with the same sound; ask her to identify that sound. Then discuss how they differ; i.e., they have the same last sound, but the first sounds are different. Why? Because they start with different letters. Explore this set of ideas for a while, then move on to the next cluster of rhyming words and do it again. Take your time, but cover all four groups of rhyming words.

o *Word Groups Having the Same Beginning Sounds*. Next – perhaps in an entirely different homework session using the same Sample Homework -- we focus on groups of words that have the same first two letters but differing last letters. There are six such pairs of words in this Sample Homework: (i) hat and ham; (ii) sat and sam; (iii) rat and ran; (iv) fat and fan; (v) mat and map; and (iv) tat and tap.[6] Go through the same process as with the rhyming words (with appropriate changes to reflect that we are now focused on the syllable sound at the beginning of the word).

o *Patience*. Remember that this is a gradual process; expect your child to be slow to learn the sound-merging skill.

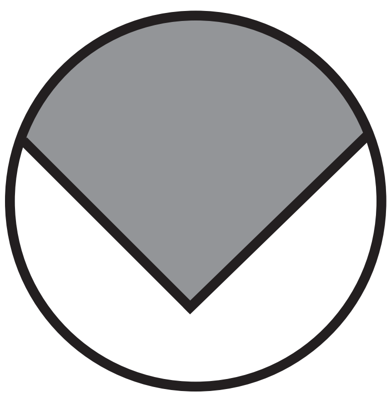
- *Fractions*. Discuss the fact that the shapes are each divided into two halves.

o *Definitional Characteristics of Fractions*. One of the abstract concepts that we have been touching upon from time to time is the idea of defining mathematical things – objects and operations – by a list of definitional characteristics. (For example, you may recall that we defined rectangles as shapes having four sides and four right angles.) Here, we can take a similar approach to fractions. Specifically, we define the fraction "½" operationally by identifying two key components to the idea of dividing a shape into halves:

■ *Number*. First, we must divide the shape into two pieces, not three, not four, but TWO; and

■ *Size*. Second, each of the pieces must be the same size.

For fun, you may want to discuss with your child the notion that two halves can be the same size even though they are not the same shape. For example, the shaded area is equal to the unshaded area in the following diagram, so the circle can properly be said to be divided into two halves (but not into two identical halves):



6 Yes, "tat" is a word; it means to make a certain kind of lace known as "tatting." An unrelated use also occurs in the phrase "tit for tat."

This same approach should be used as additional fractions are introduced in ensuing Sample Homeworks. (Of course, the first component of the meaning of a fraction, the number of pieces, will change depending on the fraction – e.g., in the case of thirds, the shape must be divided into THREE pieces, etc. -- but the second component, equal size pieces, applies to all fractions.)

o Ask your child to count the halves in each shape. Ask her to color one of the halves of each shape green. Then ask her how much of the shape – what fraction of the shape – is colored green ("One half!").

o Of course, the lines that we drew in the shapes are merely one way to divide the shapes into half; ask your child to find other ways, other lines that can be drawn to cut the shape into two equal pieces. If it helps, redraw the shape by hand, or have her do it, so that she has a clean shape to divide into half.

o Remember to do Field Work on the topic of fractions (discussed at the end of this Chapter).

6. Sample Homework #68.

• *Reading: Vowel Sounds.* In the preceding Sample Homework, we focused on words that had the same last sounds (rhyming words) and words that had the same first sounds, using groups of these words to help your child learn to merge letter sounds together into word sounds. Now, we extend this idea by keeping the first and last letters the same and changing the middle vowel – e.g., "hat," "hot" and "hit"). Again, this work is intended to help the child get a feel for the way in which letter sounds hook together to make the sound of a word. Tackle each row of words as a group and discuss the similarities and differences in the sounds of the words. Focus explicitly on how the letter sounds slide together to make the word's sound.

• *Shapes/Fractions*. Ask your child to name the shapes (as usual) and then divide them into half any way she chooses. It is important that the two halves be approximately equal in size! Have your child color in one half. Tell her that the colored part is not the whole shape, it is only half of it. Show her that there are two halves, but only one is colored. Explain that this is a fraction – less than the whole shape -- and that we write it by putting the number of colored halves on top, drawing a line, and putting the total number of halves on the bottom (i.e., one over two -- ½). Do this several times, writing the fraction "½" as you explain. Then ask your child to write the fraction "½." Finally, ask her to color two halves of the square blue, etc. Ask, "How many halves are there in any shape?" Discuss.

• *Addition: the "Call Yourself" Method.* Now that we have moved to harder addition problems, the "finger" method of addition has become cumbersome. We make your child's life easier by introducing a new, better method for doing additions, the "call yourself" method. This method can be used for any addition problem but is especially powerful for sums greater than ten.

- *Example.* Let us work with the first addition problem in this Sample Homework, "2+9." To do this problem using the "call yourself" method, your child should point to herself and call herself "9" – she should always call herself the larger number – and then count up from nine two more numbers, using her fingers to tick off the two counts ("10, 11"). The answer is eleven, the last number that she counts.

- *Another Example.* To do the second addition problem in this Sample Homework, "13+5," your child should call herself "13" and then count up five from "13," ticking the counts off with five fingers, as follows: "14, 15, 16, 17, 18." The answer is eighteen, the last number that she counts.

- *Two Steps: Call Yourself and Counting.* As you can see, the "call yourself" method has only two steps: (i) call yourself the larger number; and (ii) count up, using your fingers, the smaller number.

- *Benefits of the "Call Yourself" Method of Addition.* This new method displays the connection between counting and addition, moves the process to a more abstract level (which is important in paving the way for future skills), allows the addition of larger numbers, and also can be used (in a slightly modified form) for subtraction problems.

- *Experimenting Is Good.* Sometimes when we give a harder addition problem (such as 13+5), the child will first try it with the old finger combining method. This is OK – let her experiment and see what she can do. You will be amazed at the mental versatility of these little people!

A Comment On Our Non-Linear Path. Note that we are not following a linear path like that followed by most school systems. Instead, we are mixing together a variety of concepts (fractions, the idea of a sentence) and processes (reading, addition) with basic facts (letters, numbers). This approach is unorthodox, but has many advantages; it:

- keeps the child interested;
- gives the child a framework on which to hang facts;
- gives the child a reason to learn the facts (to aid the processes such as reading and addition); and
- matches the way a child's brain works – integrating many different types of knowledge, finding patterns and rules in a disorganized pile of material, etc.

Your child will learn, understand, and retain better if she is forced to do some of the heavy conceptual lifting (pattern recognition, abstraction of rules, organization of disparate facts into an integrated framework, etc.), as opposed to having things presented to her in a highly organized, systematic fashion. We relish disorder and rich variety of content, because it allows the child to learn the material the way that children are programmed by nature and God to learn it – and she will learn it better, and more deeply, and faster than if we teach her as if she were just a very young adult.

7. *Sample Homework #69.* Note that sentences are back. Again, as in Sample Homework #64, point out the periods and discuss the idea of a sentence. Then have the child read each sentence as a separate task. The word "I" should be memorized (obviously, like "a" it is a very easy word because it just says its name). In the open space at the bottom right of the page, ask your child to draw three circles and divide them in half different ways. Discuss.

8. *Sample Homework #71*. We continue to work on memorizing the word "the." Encourage your child to use the new "call yourself" method to do at least some of the addition problems. (If you are very lucky, she might ask if there is a "call yourself" way to do subtractions. There is! See the discussion of Sample Homework #98 below.) Meanwhile, we learn more about fractions:

- *Reading the Fraction.* A very important part of the Plan's math program is to teach math as a reading skill. We apply that focus here to the two fractions in this Sample Homework, ½ and ⅓; we "read" them just as we might read a sentence.

    o *Vocabulary: "Numerator" and "Denominator."* We begin by teaching the correct vocabulary for the parts of a fraction. The top number is called the "numerator" and the bottom number is called the "denominator." Practice these words a few times with your child, then use them unhesitatingly thereafter. So long as you refrain from sending signals that these words are too hard for your child to master, she will have no trouble at all with them. (After all, she had no trouble learning "Sponge Bob Square Pants," right?) [7]

    o *Three Steps*. We read a fraction from top to bottom in three steps. For example, to read "½" we do the following:

    - Read the top number – "one;" [8]
    - Note that the line in the middle, between the numerator and denominator, does not have a sound,[9] but instead is a "marker" telling us that we are working with a fraction and will have to use "fraction words" when we read the denominator; and

7 Or is it "Square Bob Sponge Pants?"

8 Did you notice? We said "top number" instead of "numerator!" Shame on us, and shame on you if you did not notice!

9 Unlike the line in an addition or subtraction problem, which stands for "equals." Once again, we encounter a symbol with multiple, context-dependent meanings!

- ■ Read the denominator using a "fraction word" for the number (here, "half").

o *Fraction Words*. "Fraction words" are generally the name of the number followed by "th" – e.g., "sixth," "seventh," "twenty-eighth," "one-hundredth," etc. The primary exceptions are "half" and "third," which must be learned as special cases.

• *Understanding the Fraction*. Now, we interpret the fractions geometrically. (In the second part of the Pittsburgh Plan, which we hope to compile and describe in a sequel to this book to be entitled Newton Ascendant: The Pittsburgh Plan, Part 2, participants also learn to interpret a fraction as a division problem and as a ratio.) The interpretation will depend, naturally enough, upon the interpretation of the numerator and the denominator, although not in that order! As is our custom, we start at the concrete level – with the problems from the Sample Homework – and extract general principles from our work there.

o *Problem 1: "½" of the Circle*. You may wish to approach this problem as follows:

Sample Transript: Fractions

First, read the fraction, "½," with your child. Then ask her, "What is the denominator?" When she says "two," say, "Very good! You are right! The denominator is two. And this means that we divide the circle into TWO equal pieces, because the denominator is TWO. Can you do that?" After she draws a line dividing the circle in half, you might continue, "Yes, because the denominator is TWO, we divide the circle into TWO equal pieces. What if the denominator were THREE, instead of two? If the denominator were THREE, how would we divide the circle? How many pieces, if the denominator were THREE?" [Discuss.]

Continue: "OK, well done! Because the denominator was TWO, we divided the circle into TWO pieces. What part of the fraction told us how to divide the circle?" [Do not worry if she cannot answer this. We do not expect instantaneous results, we proceed gradually, by accretion, like gentle rain falling on …. OK, OK, you get the drift! You have already heard that speech too many times!]

Continue: "We have talked about the denominator. It told us how we should divide up the circle into pieces. Now, let's talk about the numerator! What is the numerator of this fraction, '½'?" [Wait for her answer, help if needed.] "Yes, the numerator is one. Very good! What do you think the numerator tells us?" [Again, wait for her answer, and discuss it if she comes up with one.]

> *"The numerator tells us how many of those two pieces we are talking about when we say "½" of the circle. If the numerator is ONE, how many pieces are we talking about? That's right! ONE! So, since the numerator is one, go ahead and color ONE of the two pieces any color you want." [Wait for coloring to be completed.] "Very good! Now, can you point to the part of the circle that is one-half of the circle? Very good! The colored part is one-half of the circle!" [Feel free to discuss the idea that the non-colored part is also one-half of the circle.]*

o *Problem 2: "¼" of the Circle*. Repeat a version of the above process with the second problem, this time dividing the circle into four equal pieces.

o *Theory*. Now, discuss the two problems and see if your child can generalize a little bit – with your gentle guidance -- about the rules for interpreting fractions of shapes (or, as we might say, the geometric interpretation of a fraction). Do not be pushy, but you might try to make sure that some semblance of the following two general rules is floated into the ether:

■ *First Step: Interpret the Denominator.* We begin with the denominator. It tells us how to divide the shape: if the denominator is "2" (as it is in "½"), we divide the shape into two pieces; if the denominator is "3," we divide the shape into three pieces, etc. Of course, the pieces must be equal in area, and it is usually easiest to make them uniform in shape as well.

■ *Second Step: Interpret the Numerator.* Once we have divided the shape into pieces, per the instructions given by the denominator, the numerator tells us how many of those pieces are in the fraction. For example, if the fraction is "⅜," we know (from the denominator) to divide the shape into eight pieces, and (from the numerator) that we are talking about three of those pieces. How much of a circle is ⅜? Divide the circle into eight pieces, color three, and the colored portion of the circle is ⅜ of the circle. Of course, you already know this! But this is a good way to present the geometric idea of a fraction to a young child.

Interlude.

• *Child-Generated Homework Sheets.* Sammy was so excited by her progress at this point, especially in the area of reading, that she refused to take a day off, instead generating and working two homework sheets on her own. This is a good sign! If this occurs with your child, do not correct any errors. Simply praise her and pay a lot of attention to her work. Technical details will be ironed out later; the major victories already have been won: (i) your child feels empowered, she has self-esteem; and (ii) her natural curiosity and love of learning have been awakened and strengthened.

- *Do Not Correct Trivial Errors.* The preceding paragraph lays out a specific instance of a general rule: Do not correct "trivial" errors. Remember, we are pursuing cognition and understanding; the schools are excellent at teaching rote memorization, handwriting, and similar tasks. So, for example, do not spend valuable "interpersonal equity" – and risk affecting your child's enthusiasm -- to correct a backward three or a phonetically misspelled word (e.g., "kat"). These problems will sort themselves out as time goes by; we want to reward the child's progress in the critical areas of math and reading, not focus her efforts on the things that the standard school curriculum emphasizes.

9. *Sample Homework #73*. This is a significant Sample Homework, one in which we take a large step forward in reading.

- *Sentences*. Note that there are five sentences. Even before reading them, discuss why we know that we have sentences instead of just words (initial capitals, periods). And, discuss that this means the words will tell some sort of idea or story – they are connected. (We are laying the groundwork for reading by context.)

- *Unfamiliar Words.* Although most of the words are within the scope of prior work, there are a couple of curve balls: "run" (which has a vowel that is new to your child) and "ball" (which has a new sound for the vowel "a"). We can use this new sound for "a" to help build the notion that reading requires flexibility, perhaps laughing with the child as we explain that English is a funny language, and every rule is broken a lot, so that part of the game is to try different things when we are sounding words out. For example, it is fun to point out here that the "o" sound in "hot" is pretty much the same as the "a" sound in "ball."

- *Two-Vowel Rule (an "Outpost")*. We can use the word "see" to introduce one of our two important Vowel Reading Rules, namely, the Two Vowel Rule: When we have two vowels in a row, they usually (but not always) make a sound that is the name of the first vowel. Here, we have two vowels in a row -- "ee" in the word "see" (ask your child to circle the two vowels that are next to each other) – so according to the rule they will sound like the name of the first vowel , which is obviously "e." And, of course, this works. We are not really trying to teach the rule here; we are just establishing an outpost to help when we introduce the rule in full force in Sample Homework #78.

- *The Three Techniques of Reading: Phonetic; Recognition; and Context*. As your child's ability to read increases, you will notice the development of three different reading strategies, which are discussed below in order of increasing complexity and sophistication. All three techniques are very important; the Plan will give your child ample opportunity to develop skills in each.

o *Phonetic Reading*. First, of course, you will see your child sound words out (phonetic reading). This is by far the most important strategy for beginning readers, and will serve your child well throughout her life when she encounters strange or unfamiliar words. Surprisingly, although phonetic reading seems to play a critical role in the development of reading skills, it is actually the strategy that is used the least as a reader matures. Nonetheless, the Plan places heavy emphasis upon phonetic reading because it appears to be a necessary bridge to more advanced reading techniques.

o *Recognition Reading*. Second, you will see that your child memorizes words, sometimes as part of our program, as in the case of the word "the," and sometimes spontaneously because of repetition (one of our core teaching techniques). This type of "recognition reading" is very important; in fact, it is the primary reading strategy employed by adult readers. However, although the Plan does encourage memorization in some cases, this is not an overt emphasis of the Plan. Instead, the Plan lays the groundwork for recognition reading using a three-part strategy:

- *Establish An Outpost*. First, we establish an "outpost" of recognition reading by encouraging and applauding recognition reading of a small set of key words (common words such as "and," "the," "of," etc.), so that the child understands that it is OK to read a word this way, rather than phonetically;

- *Phonetic Groundwork*. Second, we emphasize phonetic reading, which appears to be a necessary precursor to more advanced reading strategies (for reasons that are not well understood); and

- *Repeated Exposure*. Third, we give the child many opportunities to read a familiar set of words that have been learned thoroughly at the phonetic level (e.g., the "at" words).

In our experience, this combination of an early outpost, phonetic preparation and repeated exposure allows recognition reading to develop naturally and rapidly.

This is just one more example of the Plan's basic philosophy at work: if a child is presented with repeated exposures to a type of challenge (such as reading), she will be far more receptive to, and in fact will often develop on her own, sophisticated strategies such as recognition reading for coping with that challenge. We "prime the pump" with the three-part strategy described above. But, as always, the real work is done by the child.

o *Contextual Reading.* There is a third, less familiar strategy that you will come to notice – namely, the use of context. How important is context in language? Well . . . .

■ Consider the word, "Stella!" shouted by Marlon Brando at the end of the movie A Streetcar Named Desire. That single word conveys vast meaning! Why? Certainly the word itself has a very limited meaning – it just names a specific individual.[10] And there is little syntactic meaning in a one-word sentence! So, if the meaning does not come from the word itself, or the structure of the sentence (i.e., the syntax) in which it occurs, from whence doth such meaning spring? The answer is simple; the word "Stella" derives its power and its vast content from the context in which it occurs – i.e., from the background facts established by the entire movie.

■ Or, consider Lou Gehrig's statement, in his farewell speech at Yankee Stadium, that he was "the luckiest man on the face of the earth." This statement on its face is scarcely memorable; it derives its power from the context, the fact that the great first baseman, the indomitable "Iron Horse," made the statement while suffering from an incurable and terminal disease (amyotrophic lateral sclerosis, now known as "Lou Gehrig's Disease").

How will this powerful strategy, this idea of contextual reading, surface in the homework sessions? Your child will begin to use contextual clues in her reading on her own, without any prompting from you. Contextual reading occurs, for example, when your child expects a certain word to occur and "fills in the blank" in a sentence by reading the expected word instead of looking carefully at the text in front of her. This is a good strategy, and usually produces the correct result, especially in experienced readers. You really only notice this strategy in operation when an incorrect word is selected and read. For example, if the text reads "The girl slid down the slope", your child may well read, "The girl slid down the slide." This kind of error often occurs when the word that is misstated is a new or unfamiliar word. Although you should point out these types of errors, they are actually a good sign, because they suggest that your child is beginning to read at an advanced level. It is this type of reading, contextual reading, that allows a person to process large amounts of material rapidly – perfect for cramming for finals in college or reading The National Enquirer in the checkout line.

• *Sample Homework #73 Encourages Use of All Three Techniques*. Obviously, we are putting most of our energy into phonetic reading, and it is that technique which we expect will be of primary use to your child in reading the five sentences in this Sample Homework. However, by now your child may also recognize some of the words. And, the sentence "I can hit the ball" provides a great opportunity for the use of context to help the child figure out a word with an unfamiliar vowel sound for the "a." Watch and see if your child uses the contextual clues provided by the word "hit" to help her figure out the word "ball."

10 In fact, absent context, the word "Stella" does not even do this; instead, it just names an unspecified member of the large group of women who are named Stella!

• *Reading the Sentence as a Whole.* After your child has sounded out the words in the sentence, ask her to read the sentence as a whole, and repeat this process until she can read it fairly smoothly. This technique prompts a form of recognition reading.

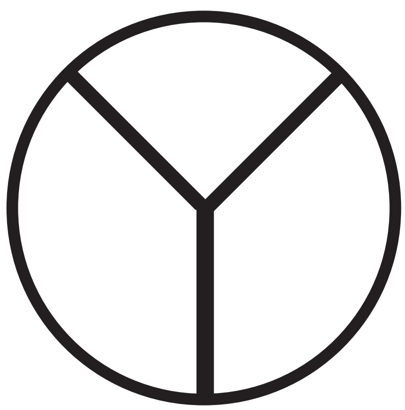
• *Zero Rules.* One of our more subtle math goals is to help the child learn to use rules to solve problems, rather than relying on brute force in every instance. The main set of rules currently at our disposal relate to zero:[11]

- o zero plus anything equals that thing (and vice versa);
- o anything minus zero equals that thing (but not vice versa); and
- o anything minus itself equals zero.

You might encourage your child to think of the subtraction problem "1-1=" in these terms.

Another rule that you may want to introduce is the idea that, for example, 4+3 equals 3+4, or 9+1 equals 1+9. This rule, known technically as the "commutative law" of addition, can ultimately help reduce the child's memorization workload. It might be fun to talk a little bit about why the same rule does not work for subtraction. [12]

• *Fractions: One-Third.* Read and interpret the two fraction problems just as in Sample Homework #71 above. Note that the best way to divide a circle into three equal pieces is to draw a "Y" inside it, like this:



11 In a moment of creative inspiration, we named these rules the "Zero Rules."

12 For the mathematically sophisticated reader, the addition/subtraction situation is more symmetric than appears from the above discussion. In fact, addition and subtraction are the same; subtraction is just the addition of a negative number. From this perspective, subtraction is commutative, in the sense that 7-4 is actually 7 + (-4), which does in fact equal (-4) + 7.

10. *Sample Homework #74.* We introduce the new number "100" as a fun outpost in the "land of large numbers," and also list the fraction "½" in our rows of numbers, allowing a discussion of the idea that ½ is a number, just like "1" or "100," and is not just a geometric concept. (The Field Work involving fractions and groups of coins will also help support this broader idea of a fraction.)

- *"Girl."* For many children, the "ir" sound in "girl," which requires the production of an "r" sound very deep in the throat, is the most difficult sound of all to produce. Here, it is introduced merely as an "outpost," an early introduction to this hard sound. (In fact, Sammy had a great deal of trouble with this sound. We will discuss how to handle such situations later in the book in connection with Sample Homeworks showing our remedial approach; at Sammy's age at the time of this Sample Homework – barely four – the inability to produce the "ir" sound was quite normal and was not cause for special action.)

Our discussion of the "r" sound here reminds me of two rabbit-related stories involving Plan participants:

o I was driving two young Plan participants to pre-school. The younger child, my son, pointed to a rabbit in a nearby field and squeaked excitedly, "Look, Suzie, a bunny!" Suzie, a neighbor's daughter, replied with an air of great superiority, befitting her advanced age of four, "only babies say 'bunny.' That is a WABBIT!"

o One of our five-year-old participants had just begun kindergarten and was having trouble paying attention to the teacher. We suspected the problem was that she was bored; the teacher was teaching the class, laboriously, about "Miss A" for a week, then turning to "Mr. B," etc., while the five-year-old was reading chapter books, including, for example, <u>Aesop's Fables</u> (you will see the relevance of this in a minute). But, at the request of the teacher, the five-year-old was taken to a laboratory for a hearing screening. As part of the procedure, the technicians tried to elicit a word-initial "B" sound from the girl – in particular, they tried to get her to say the word "bunny." To that end, they showed her a picture of a rabbit, and the following conversation ensued:

Technician: "Please tell us what this animal is, sweetie."
Girl: "A rabbit."
Technician: Very good, honey. But what is another name for a rabbit?"
Girl: "Hare."

- *Outpost: The "Silent 'e' Rule."* In Sample Homework #73, we briefly introduced the first important vowel reading rule, the Two-Vowel Rule. Now, with the word "like," we introduce the second key vowel reading rule, the "Silent 'e' Rule" (also know as the "Boss 'e' Rule").

o *Technical Statement of the "Silent 'e' Rule."* Under this rule, an "e" at the end of a word is often silent, making no sound, and instead has the indirect effect of telling the immediately preceding vowel to have a long sound; to "say its name." Because the "silent e" tells the preceding vowel how to sound, we sometimes refer to the "silent e" as the "boss e." The rule usually applies only if the "e" and the preceding vowel are separated by a single consonant.

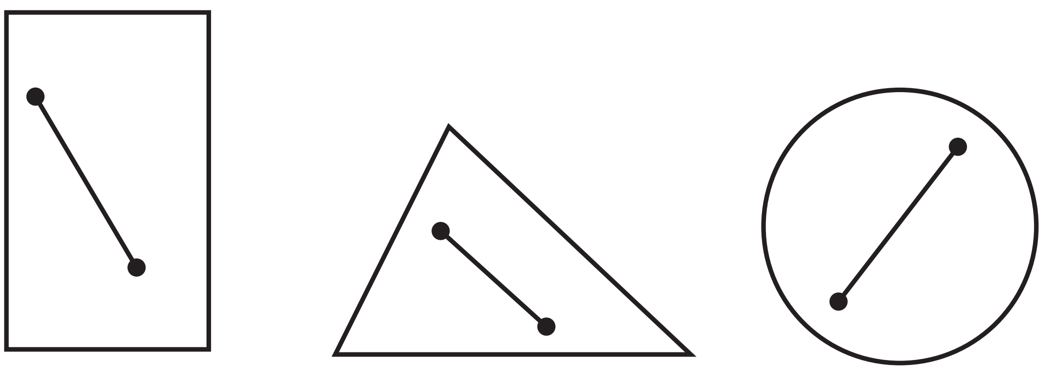
o *Discussion of the "Silent 'e' Rule" in the Homework Session*. Of course, you should not discuss the above technical statement of the Silent e Rule with your child. Instead, merely point out the rule, and explain in concrete terms how it works in the word "like." The main concrete points to convey are (i) that the "e" at the end of the word "like" is silent, and makes no sound at all, (ii) that instead the "silent e" bosses around the "i," telling it to sound like its name, and (iii) that there is only one letter between the "i" and the "e."

• *Outpost: Plural "s" Rule*. The word "cats" allows us to introduce the rule that most nouns are made plural by adding an "s" at the end. Ask your child to read the word, and then ask what it means. Ask her how "cats" is different than "cat;" i.e., what is the effect of putting the "s" at the end of the word? Discuss other examples – "dogs," "hamsters," etc. See if your child can figure out the rule.

• *Shapes.*

o *New Shapes*. Before working on dividing the shapes, ask your child to identify the shapes (as usual), and discuss the two new shapes, a "lemniscate" [13] (which can also be described as a "figure 8") and a "cube" (which can also be described as a "box").

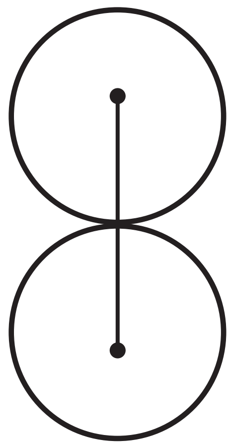
■ *Lemniscate*. The most interesting thing about the lemniscate is that, unlike every other shape that we have studied so far, parts of the interior are separated from each other by the shape's boundary. What does this mean? Well, consider a square. Any two points in the interior of the square can be connected by a line (straight or curved) that does not pass through the boundary of the square. This is also true of a circle, triangle, rectangle, and many other simple shapes. E.g.:



13 "Lemniscate" is a great word that you should consider teaching to your child. The most famous examples of a lemniscate are the number "8" and the sign for infinity: "∞."

Note that this is not true just for some interior points; it is true for every pair of interior points. They can all be connected without going across the boundary of the shape. (For concave shapes, such as a crescent moon, the connection might have to be a curved line, but that is OK; it is still a line that does not pass through the boundary.)

On the other hand, this is not true of a lemniscate! It is easy to see that there are certain pairs of interior points that can be connected only by a line passing through the boundary of the lemniscate:

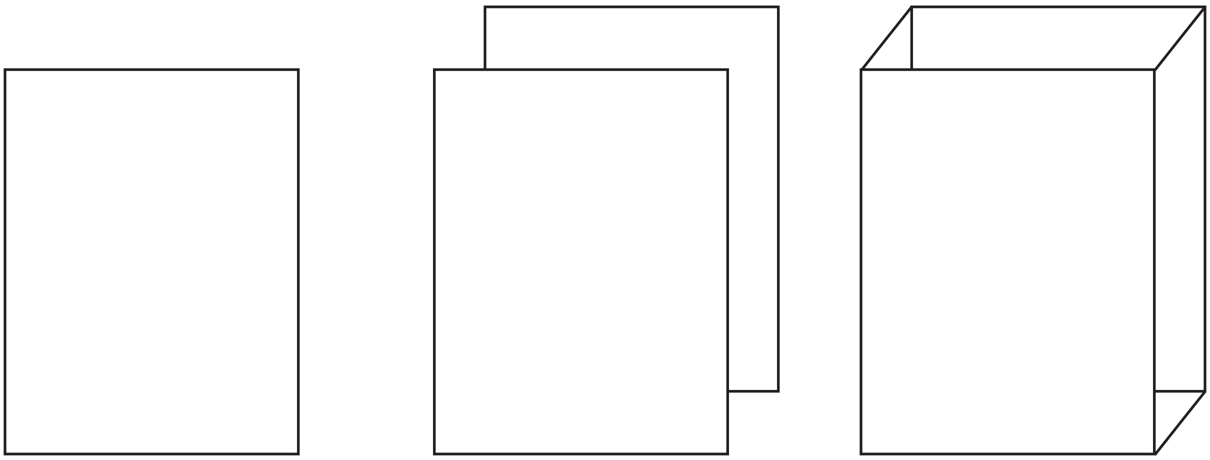


"We are not really heading anywhere specific with this topic; we are just enriching your child's intellectual environment . . ."

Discuss these ideas with your child – feel free to draw diagrams on a blank sheet of paper and play with them. We are not really heading anywhere specific with this topic; we are just enriching your child's intellectual environment, throwing out some ideas that are related to important topics (e.g., "simply connected" sets) found in topology.

■ *Cube.* The cube is also fundamentally different than any other shape we have encountered. Why? Because it is a three-dimensional shape, not a two-dimensional shape like a circle or a square. A two-dimensional shape can be pressed flat onto a table, or pasted flat onto a wall. It had a length and a width, but no height. On the other hand, a three-dimensional shape – such as a sphere or our new friend the cube -- cannot be pressed flat on any surface, because it also has a third dimension: height. Discuss these ideas with your child, focusing especially on how in this respect all our prior shapes are like each other, but the cube is different. One telling way that this difference manifests itself shows up when we try to divide shapes by drawing a line – we cannot divide a cube in half by drawing a line through it. We need something more (i.e., a plane, like a sheet of paper) to slice a three-dimensional shape in half.

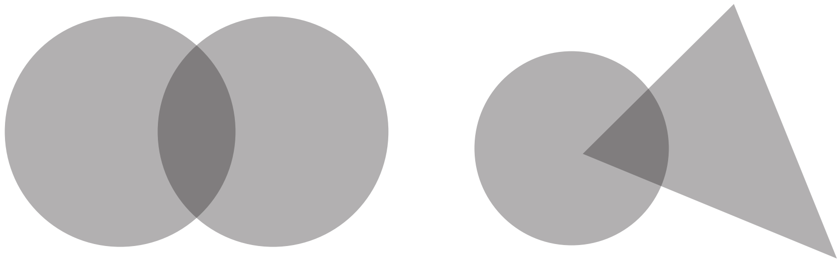
■ *Just For Fun.* Just for fun, teach your child how to draw a cube. As you probably know, the easiest way to do this is to (i) draw a square, (ii) draw the same square slightly up and to the right of the first square, and (iii) connect the three visible vertices (and erase the interior line segments if you wish):



o *Fractions: Dividing Shapes In Half.* Ask your child to divide the shapes in half. The lemniscate can be divided either with a vertical line (separating it into a "3" and a "backward 3") or a horizontal line, separating it into two circles. It is probably worthwhile to try to get your child to do it both ways; help her if she needs a hint or two. NOTE: As discussed above, the cube is a special case – it cannot be divided in half with a line!

11. ***Sample Homework #75***. Here, we ask your child to produce (rather than just recognize) two words.

12. ***Sample Homework #76***. Building on our prior work relating to the ideas of boundary and interior, we introduce the idea of an intersection of two shapes – i.e., the area inside both shapes, or the area "that is part of," or "belongs to," both shapes. Ask your child to color the intersection – the area inside both shapes – red, and the ask her to color the area that is inside only one shape blue, like this (we are limited to using shades of gray, but you get the idea!) :



Ask her to point to the area that is outside all of the shapes – the exterior of the figures.

13. ***Sample Homework #78.***

- *New Letters.* We introduce our last two letters, "q" and "d," and begin a theme that will recur frequently in future Sample Homeworks: we use the row of letters to work on distinguishing the most confusing letters, namely b, d, p, q, g, h, and n.

- *New Numbers*. We introduce "12" as well as our first "teen" numbers: 14; 16; and 18.

  o *Even Numbers.* Note that these four numbers are all even numbers, which for us at this time means they are on our list of numbers when we count by twos (2, 4, 6, 8, etc.), as discussed in the Field Work.

  o *Reading Teen Numbers*.

  ■ *Inductive Approach to the General Rule*. Teach the general rule for reading two-digit numbers starting with "1" inductively. Start with "14" and read it by saying "Four . . " (pointing to the four) ."…Teen!" (pointing to the one). Then, do the same with "16" and "18." Ask your child if she can guess the rule for reading teens. (Of course, the rule is simple; you read a teen number by reading the second number and then saying "teen.") Try to see if your child can guess this rule by herself; see if she can read "17" or "19."

  ■ T*here Are As Many Exceptions As There Are Numbers That Follow the Rule!* Now, have a little fun with your child. You have already worked with her before on the idea that English is a strange language, where every rule has exceptions. Well, this rule has as many exceptions (five: 10; 11; 12; 13; and 15) as it has adherents (also five: 14; 16; 17; 18; and 19)! Discuss this. For example, thirteen and fifteen can be made to fit this rule with a little bit of stretching, and, after all, kids are very good at stretching rules! On the other hand, there is no way of talking ourselves around "10," "11" and "12;" these numbers are flat-out, no-excuses exceptions to the rule.

- *The Beginning of a Long Program of Reading Consolidation.* This Sample Homework marks the beginning of a significant period of consolidation of important reading skills, particularly the Two Vowel Rule, the Silent "e" Rule, the memorization/recognition of words, and the reading of consecutive consonants. We use this Sample Homework to go into these three topics in some depth, and will continue such in-depth work for a significant time. We are on the very edge of independent reading, one of the landmarks in your child's entire life!

A Note On Our Objectives.

■ *Big Objective: Independent reading ASAP.* Our principal high-level reading objective is to have your child begin reading on her own as soon as possible. Once this occurs, it will not be necessary for us to focus on reading in the homework sessions to any great extent; reading will take care of itself so long as you provide a rich variety of reading material for your child (including comic books, etc.).

"Our principal high-level reading objective is to have your child begin reading on her own as soon as possible."

■ *Continue Outside Reading Support Activities.* You should continue to have your child read to you even after she is an independent reader. The homeworks are not intended to be a substitute for other parent-child activities! Instead, we ask that you make time for the homework sessions by eliminating a corresponding amount of time from passive activities such as television, computer games (even educational computer games), etc. In particular, it is important that you continue to read with your child at every opportunity, and also try to find books that she will enjoy looking at or reading on her own. Do not pressure her to read; simply read with her, and give her the opportunity to read. You are still some distance away from having a reading child, but you are much closer than you would have been absent your steady commitment to the homework sessions.

■ *Homework Sessions: Shift From Reading to Math.* Because the joy of reading is so compelling to an early reader, and because schools are fairly good at encouraging early readers, we will eventually shift the focus of our homework sessions to math, where the need for help is far greater. Our theory for this shift in emphasis down the road (typically when the child becomes an independent reader) is that (i) schools are not very good at teaching math, especially to early achievers, and (ii) the expectations for math achievement of young students are set very low, meaning that it is easy to bring your child to a point where she can excel in math measured against both the expected academic norms and the performance of other students. Thus, we can build a lot of self-confidence and energy by focusing just a little bit of time on math.

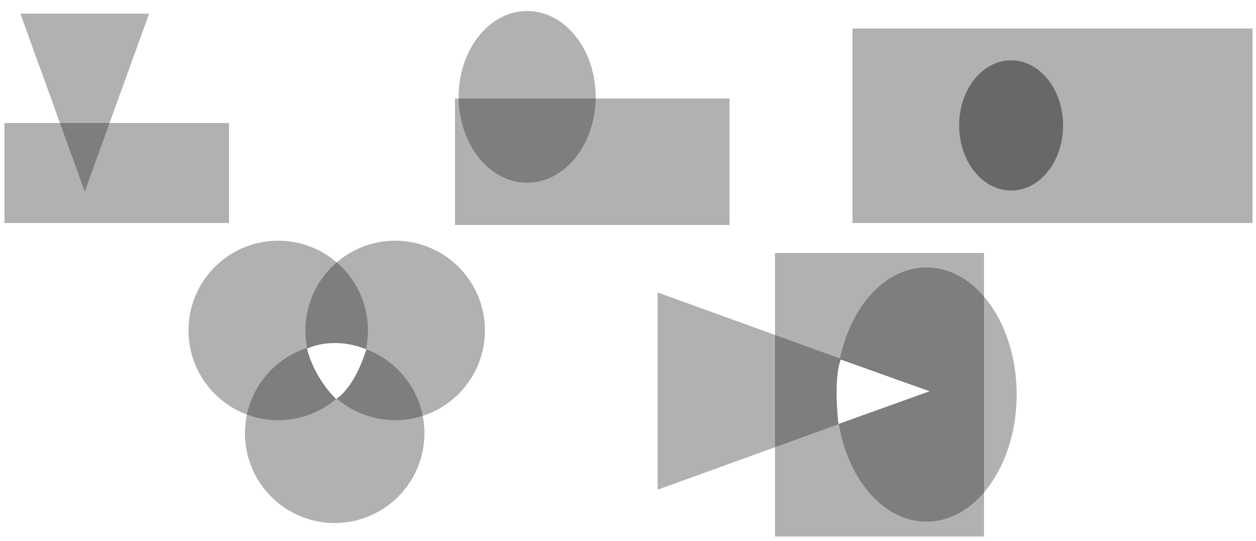
o *The Two-Vowel Rule.* The first row of words in this Sample Homework calls into play the Two Vowel Rule, which was introduced briefly in Sample Homework #73. This rule states that when two vowels occur next to each other in a word, they (usually) make the sound of the name of the first vowel – i.e., they sound like the "long" form of the first vowel. Thus, "oa" and "oe" usually sound like the name of "o," "ea" and "ei" usually "say the name" of "e," etc. Of course, this rule is loaded with exceptions, but it still a handy tool for a beginning reader.

■ *Teach By Induction*. As always, we recommend that you teach the rule by induction – the fact that the rule was mentioned briefly in an earlier session or two might well help the inductive process. Start with the word "soap" and have your child circle the two vowels in the middle of the word ("oa"). Then, work with her on various readings until you and she arrive at the proper pronunciation of "soap." Ask leading questions to elicit the idea that the circled vowels sound like the name of the first vowel. Go through the same procedure with the rest of the words in the first line of words, always having your child circle the two adjacent vowels. At the end of that line of words, discuss the handling of two vowels with your child and, one way or another, get a general statement of the rule out on the table. (It is best if your child says it, but it is OK if you say most of it and your child fills in a blank or two.) However the rule finds its way to the surface, see if your child can repeat it back to you.

■ *Many Exceptions.* Point out to your child that there are many exceptions to this rule (e.g., "out," "good" and "poem").

o *The Silent "e" Rule.* The second row of words in the Sample Homework calls into play the Silent "e" Rule, which was introduced briefly in Sample Homework #74. This rule states that (usually) when a word ends in an "e," and there is only one consonant between that "e" and an earlier vowel, two things will happen: (i) the earlier vowel will "say its name" (i.e., have a long sound); and (ii) the "e" at the end of the word will be "silent;" i.e., it will have no sound at all. Again, it is recommended that you teach this rule inductively, using the words in the second row of words, and emphasize that there are many exceptions to this rule.

• *Advanced Intersections.* Here we have five very interesting intersection diagrams! For the top three diagrams, use the same coloring instructions as in Sample Homework #76 (red for the intersection of two shapes, blue for the interior of only one shape). The third diagram (a circle contained entirely inside a rectangle) is a little tricky; remember that the intersection is the area inside both shapes; i.e., it is the entire circle! In the bottom two diagrams, both involving three shapes, you should ask your child to color: (i) the intersection of three shapes (i.e., the area that is inside three different shapes) yellow; (ii) color the intersections of only two shapes red (as above); (iii) color the area that is inside only one shape blue (as above); and (iv) point to the area that is not inside any shape at all (again, although we are limited to using shades of gray, the following illustrations should be fairly clear) :



14. ***Sample Homework #80***. Ask your child to try to write the word "not" in the open space at the end of the first line of words; work with her in sounding it out to determine what letters to write. (Note that "not" appears later in the Sample Homework.) When your child reads the short four-sentence story:

- *"Said" Notation*. Discuss with her the "said" notation in the second sentence (said, "...."); this will occur frequently, and is something she should memorize.

- *Exclamation Point*. Also discuss the exclamation points in the second sentence; explain that they mean the sentence should be read in an excited manner.

- *Reading for Comprehension*. After your child has read the story, ask her to tell it to you without reading it (here, we are working on reading with comprehension, rather than just phonetically).

- *Recognition*. There are many words in the story that your child should be able to read by recognition rather than sounding out. Discuss this with her at the outset.

Finally, discuss with your child the idea that three of the math problems (3+0, 4-0, and 8-8) can be solved using the Zero Rules.

15. *Sample Homework #81.*

- *Sentences*. Here, in the sentences, we continue to work on the Two Vowel Rule and the Silent "e" Rule. And:

    - *"Apple:" Consecutive Consonants.* We also introduce a difficult two-consonant word (having two, or in this case three, <u>consecutive</u> consonants), "apple." Ask your child to take her time sounding this word out – it is tricky, because (i) she has to learn that two consecutive identical consonants ("pp") just have the sound of one consonant and (ii) she has to see that the Silent "e" Rule does not apply, even though the "e" is silent. See if your child can tell you why this rule does not apply (because there is more than one consonant between the "a" and the final "e"). The hardest part of reading the word "apple" is figuring out how to slide the "p" and "l" sounds together. Tell your child to slide the "a," "p" and "l" sounds together and listen carefully to see if she can guess what the word is.

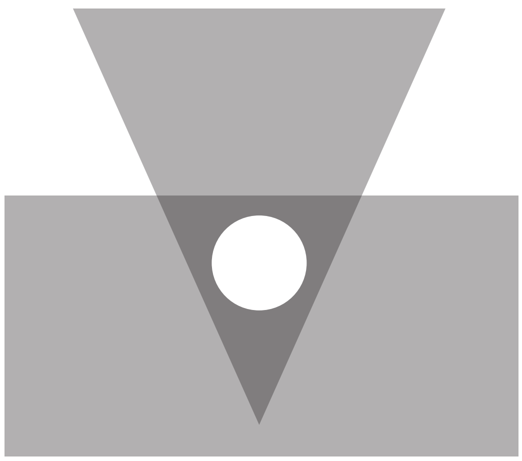
    - *<u>Memory Words</u>.* Your child should understand that the words "I" and "to" are memory words, just like "the" – i.e., words that should be recognized rather than sounded out. Discuss the funny "o" sound in "to" and explain that one of the many funny things about English is that vowels can have many different sounds, so when we come across an unfamiliar word sometimes we have to try different sounds for the vowels and listen to guess what the word might be.

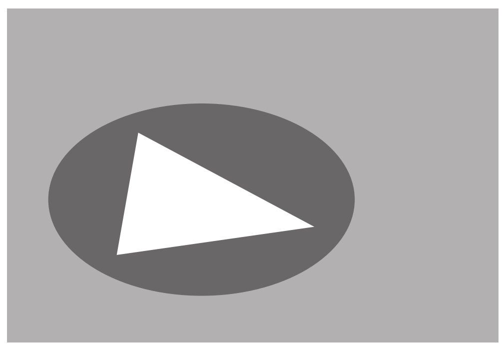
o *Read Instructions.* For fun, see if your child can help you read the instructions for the shapes. Point out that "and" is another memory word that should be recognized rather than sounded out.

• *Math Problems*. By now, your child should generally be using the "call yourself" method, described first in Sample Homework #68, for all addition problems. Point out that 8-8 can be done using one of the Zero Rules.

16. *Sample Homework #82.* Use the pairs of words to reinforce the Silent "e" Rule. Note that we introduce four new numbers: 13; 17; 19; and 20 (our first "outpost" into the broader world of non-teen two-digit numbers). Our interesting new words are "hungry" (a consecutive consonant word with a difficult "r" sound) and "good" (introducing the weird two-vowel pair "oo," which can make two principal sounds, neither of which satisfies the Two Vowel Rule – "oo" is a big exception to that rule, which is OK, that is what makes reading fun).

17. *Sample Homework #83*. Here, we read the fractions (which are handwritten just for a change) and then do our standard work on the intersections (which can be a little tricky when one figure is completely contained in another, especially as in the second intersection diagram). As in Sample Homework #78, ask your child to color: (i) the intersection of three shapes yellow; (ii) color the intersections of only two shapes red; (iii) color the area that is inside only one shape blue; and (iv) point to areas on the page that are not inside any shape at all (again, we are limited to using shades of gray, but hipefully the idea is clear) :





18. *Sample Homework #84.* Discuss the word "do" (another memory word) and its similarity to "to." Your child should work very carefully and methodically on the word "banana;" there is nothing hard about it except for the many syllables. It is a great word for introducing the patience and step-by-step approach that is required to sound out longer, multi-syllable words.

19. ***Sample Homework #85.*** Between the word lists and the sentences, this Sample Homework has seven memory words: "and;" "me" (a new word); "he" (also new); "to;" "do;" "I;" and "the."

20. Sample Homework #86. We formally introduce our last teen number, 15.

21. ***Sample Homework #87.*** "Go" is another memory word; also, focus on the "oy" sound in the words "toy" and "toys." Discuss whether "y" is acting as a vowel or a consonant here (I am not sure either!).

- *Shapes and Symmetry.* Ask your child to divide all of the shapes except the triangle into fourths; discuss why it would be harder to divide the triangle into fourths. (The problem with the triangle is that unlike the other figures it does not have two perpendicular axes of symmetry. Of course we do not expect your child to produce this idea, or even understand it, but it does not hurt to just discuss it a bit in a gentle, two-detectives-working-on-a-case way.) If you have the time and feel like it, it is fun to take this a little further and discuss the ideas that (i) there is no easy way to divide the triangle into fourths, (ii) there are two easy ways to divide the rectangle, square and square-diamond (aka "rhombus")[14] into fourths (diagonally and length-width)[15] , and (iii) there are an infinite number of easy ways to divide the circle into fourths (any pair of perpendicular diagonals). No need to reach any grand conclusions; just draw a lot of shapes and play with the ideas. (Of course, the deep underlying concept here is that of symmetry.)

- *Fractions*. Read each fraction and then ask how many of them there are in a shape (e.g., "How many thirds are there in a whole shape?"). Discuss the connection between the answer to this question and the denominator (they are the same). Use the words "numerator" and denominator" frequently and make sure your child understands them.

- *Greek Letters.* The Greek letter grades for Sammy were now routinely drawn from the following: α (alpha); β (beta); γ (gamma); θ (theta); ϕ (phi); ψ (psi, usually called "spy" by our conspiratorial youngsters); π (pi); Θ (capital theta – an "H" inside a circle, which might make a good uniform logo for a superhero named "Halibut Girl," who can shape-shift into a halibut at will); Φ (capital phi); and Ψ (capital psi or "SPY").

22. ***Sample Homework #88.*** See if your child can use context to help her figure out the word

14 I suppose by now you can guess that I think you should teach this cool word to your child – it is so much neater than "Fuzzy-Wuzzy," etc.

15 The equality of the areas of the four triangles produced by diagonal cuts may not be obvious, especially in the case of a long, narrow rectangle. Showing this equality (using the formula Area of a triangle = ½ Base x Height) turns out to be a good exercise for more advanced participants.

"Meow." Also, on a more general note, remember to feel free to hand-copy the Sample Homeworks, modifying them to suit your child's particular situation. There is something very nice about the atmosphere created by using a handwritten homework sheet – just you and your child, sitting there with pencils!

23. ***Sample Homework #90.*** Read the new number "23" by pointing to the "2" and saying "twenty . . ." then pointing to the three and finishing " . . .three." Do the same with "25," then see if your child can guess how to write "28" or "26."

24. ***Sample Homework #92***. Use the same approach as we introduce five more new "twenties" numbers. By this point, your child probably will have caught on to the trick of naming two-digit numbers, and acquisition of this type of knowledge should proceed relatively painlessly – so henceforth we generally will not go into detail about the introduction of new two-digit numbers; you know how to do that now!

25. ***Sample Homework #95***. "He" is yet another memory word.

26. Sample Homework #96. Here we introduce words ending in "ing," which will pave the way for reading the more common use of "ing" (changing a verb into its gerund form, e.g. hit → hitting). Note that the shapes include an infinity sign, "∞," the most famous of the lemniscates! Discuss with your child the idea of infinity, stressing that it is not a number, it is <u>not</u> the "biggest number;"[16] it is an <u>idea</u>, the idea of getting bigger without any stopping or limit.

27. ***Sample Homework #98***. Here we introduce the first "thirties" numbers (teach them just as you did the "twenties"), and again throw in "100" as an outpost in three-digit territory. Discuss the word "little" in the same way that you discussed "apple" back in Sample Homework #81.

- *Parabola.* Teach your child the word "parabola," perhaps asking her which letter looks like a parabola ("U"), and then discuss the idea that a parabola is not "closed;" i.e., it has an open end; it does not really have a well-defined "inside" or "interior." Discuss whether it makes sense to try to divide a parabola in half; can you divide an open shape, without a closed boundary, in half? By now, you know that the answer is not really that important; it is the discussion that matters!

"By now, you know that the answer is not really that important; it is the discussion that matters!"

- "*Call Yourself" Subtraction*. We now have a subtraction problem, "13-4," that is very difficult to solve using our finger method of subtraction. Allow your child to try the problem using existing techniques; whether or not she gets it, it will be difficult. This a motivational moment; now is the

16 If it were a number, you could just add "1" to it and make a bigger number!

time to teach the more efficient "call yourself" technique of subtraction (similar but not identical to the "call yourself" technique of addition).

- o *Teach Inductively, By Examples.* The "call yourself" method of subtraction involves backward counting. For example, to do "nine minus four" you start by pointing to yourself and calling yourself "nine." Then, since you are subtracting "4," you will count backward four numbers, "eight, seven, six, five", each time raising a finger. When you are holding up four fingers, the number you are subtracting, you stop, and the number you just recited is your answer. To do "13 – 4," call yourself "13" and count backwards "4" numbers (ticking them off one by one on four fingers): "12; 11; 10; and 9." The last number, "nine," is the answer.

- o *The Method, Formally Stated*. To use the "call yourself" method for subtraction, call yourself the starting (larger) number and then count backward, raising a finger for each count, until you are holding up the number of fingers that you are subtracting. This will mean that you have counted backward the number you are subtracting.

The backward counting is by rote, and your child's attention should be focused on her fingers, so that she knows when to stop counting. As you can see, this technique, which is very powerful for a young child, depends importantly on having mastered counting backward (blast-offs) and one-to-one correspondence (counting exercises), both of which have been practiced for a long time now.

28. ***Sample Homework #99***. Encourage your child to solve "10-10" by using the Zero Rules.

29. ***Sample Homework #100***. Focus on the question mark at the end of the first sentence; discuss it, and help your child read the sentence with the proper "question" inflection. Perhaps she will find the context of the first sentence useful in deciphering the difficult word "thirsty" in the second sentence. Spend a little time talking about the sound of "th," referring to "the" as an example. [17]

30. ***Sample Homework #101.*** This was going to be a day off, but Sammy insisted on doing a homework session, so we did a very quick one.

31. ***Sample Homework #102.*** We use the names of two friends; at the age of four, many children have some familiarity with the spelling of their friends' names, so this is one way to prompt recognition reading. Note the three math problems involving the Zero Rules.

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17 It may surprise you to learn that there are actually at least three different pronunciations of "th" in English: (i) the soft "th" sound, as in "there;" (ii) the hard "th" sound, as in "thing;" and (iii) the aspirated "th" sound (with a little bit of hissing) as in "bath-house." The word "the" actually has varying "th" sounds depending upon usage. For example, the soft "th" sound occurs when the "e" in "the" has a soft sound, as in "I want to eat the peach." The hard "th" sound occurs when the word "the" is emphasized with a long "e" sound, as in "This is THE best coffee I have ever had!" Obviously, one does not need to be aware of these distinct pronunciations at the conscious level in order to produce them correctly, one more example of the mysterious power of our innate language learning mechanism!

32. ***Sample Homework #103.*** The two sentences use the exclamation point to deliver commands; these are imperative sentences (commands). Discuss this concept with your child, focusing on the difference between an excited exclamation (e.g., "I am happy!") and a command.

33. ***Sample Homework #104.*** This sentence has a great deal of punctuation. Review the "said, ':....'" construction, then focus on the interior comma and discuss its effect. Also point out and discuss the use of the apostrophe-ess to show possession ("Kara's"). Finally revisit the question mark.

34. ***Sample Homework #106.*** This Sample Homework introduces two important ideas:

• *"Ing" Words.* Go through the list of "ing" words one by one, discussing in each case how the word was made by adding "ing" to the end of a short "action" word (i.e., a "verb" – do not be afraid to use that term!). In each case, the vowel in the main word does not say its name, and in each case there are two consonants right before the "ing" – in fact, if the original word does not have two word-ending consonants, we double the last consonant so that there are two consonants before "ing!" As you discuss all of this with your child, speculate with her about what you would do if you wanted to have the main-word vowel say its name, as in "hating" or "liking." See if your child can guess that in such cases we do not double the last consonant. As always, this is no big deal; we are not trying to teach this complicated rule; we are just giving the child a lot of examples and discussing them, allowing induction to begin its slow but brilliant process.

• *What Is a Consonant?* At this point, it might be a good idea to check and make sure that your child understands the idea of a "consonant." Probably the best working definition for our purposes is "any letter that is not a vowel."

• *Two-Digit Addition*. Now, we introduce the addition of two-digit numbers not involving carrying or regrouping: "10+11" and "12+14."[18] Of course, as usual, we write the addition problems vertically in the Sample Homework to facilitate the "adding by columns" approach we will be discovering in this session.

o *First, Dangle the Bait*. First allow your child to try (and perhaps solve) these problems using the existing techniques. For example, to solve "10+11," your child may call herself "11" and then count up "10" digits – which is not that bad; after all, she has just enough fingers to do the job! To solve "12+14," she may try to call herself "14" and count up "12" digits. This is harder; she only has ten fingers. When Sammy confronted this dilemma, she frowned for a moment and then said earnestly, "Mommy, can I borrow two of your fingers?" These are excellent efforts, and should be praised. But, you

18 Note that a problem such as 45 + 36 would not be appropriate yet, because the first task, adding 6 + 5 in the first column to get 11, involves "carrying" a "1" to the 10's column. We will teach this soon, but not yet.

should go on to discuss that it is hard to handle these problems with our existing techniques. Something more is needed . . .

o *Then, Set the Hook*. Having motivated your child to be receptive to a new technique, you should now introduce it! Of course, that technique is simply to use our old techniques to add the two columns separately, one at a time. For example, we solve "12 + 14" by adding 2 + 4 in the first ("ones") column, getting 6 and writing it down under that column. Then we tackle the second ("tens") column, adding 1+1 to get 2, and writing the 2 down under that column. Finally, we read the answer, not as two separate one-digit numbers ("two-six"), but as a single two-digit number ("twenty-six"). BUT . . . you know what comes next, right? Do not teach this technique to your child; help her discover it. After your child has tried to solve these difficult problems using the finger method, say, "Sammy, that was very hard, wasn't it? Let's try something new; let's try a trick. Maybe it will work; maybe it won't. But let's try it. What do you think? Shall we try?" Then rewrite the problem and ask her to add the two columns separately, gently guiding her through the process. Ask her to compare this answer to the answer she derived using the old techniques. Allow her the moment of discovery; they are the same! Maybe the easy new technique works! Try it again with the next problem.

In one fell swoop, we have introduced the idea of adding columns separately! Sure, your child does not yet understand the theory (base 10, etc.), but that will come, and when it does she will have a few concrete hooks – and a lot of inductive experience -- upon which to hang that theory.

- *Self-Esteem*. Can you see that your child is earning her self-esteem?

35. ***Sample Homework #107.*** We introduce Sammy to some Pulitzer-Prize-level poetry. Of course, rhyme schemes provide great contextual clues to the sounds of unfamiliar words![19]

36. ***Sample Homework #108***. We now assume that your child understands the rules for reading two-digit numbers, and feel free to introduce them willy-nilly! The sentence sprang from a real-life experience earlier that day. After Sammy's four older siblings had each been asked what they wanted to be ("paleontologist," "geneticist," etc.), Sammy's mother turned to her and said, "And Sammy, what do you want to be?" Without missing a beat, Sammy replied, "A DUCK!" She received such a great reaction we thought we might as well capitalize on it in the homework session. Note that we introduce a new, small rule: "wh" sounds like "w" (unless you are an Ivy Leaguer, in which case "wh" has a hint of extra breathiness to it).

37. ***Sample Homework #109.*** Note the plural "s" in "stairs" and the new "ch" sound in "chair." We

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19 In later stages of the Pittsburgh Plan, the beginning of most Sample Homeworks consists of a reading from the collected poems of Robert Service (known best for "The Cremation of Sam McGee").

have two new memory words, "you" and "your." These words provide a good opportunity to discuss the consonant versus vowel sounds of "y." When you get to the shapes, ask your child to read the "color" words above the shapes (this provides a valuable contextual clue!) and then color each shape accordingly. You might also discuss how the two addition problems "14+3" and "14+13" are similar and how they are different. By now, our Greek letter grades include the following additional Greek letters: ν ("nu"); ζ ("zeta"); and Σ (capital "sigma").

38. *Sample Homework #112*. Use the word "two" as an opportunity to discuss the weird trio, "to," "too," and "two." After all of our work on the various Zero Rules for addition and subtraction, you will probably be as excited as I was [20] to hear that on the original homework sheet Sammy hand-wrote the last subtraction problem, "0-0" without any prompting or parental suggestion at all! This was a great moment; Sammy took the Zero Rules and extrapolated them to their extreme conclusion – the paradigm of a zero rule! This was an ice-cream moment!

39. *Sample Homework #113*. Poetry is always a good way to teach – after reading "How now brown cow?" Sammy never forgot the sound of "ow!"

PART 2: Field Work For Phase 4.

Field Work is still important; it should include the following (many of which can be done while riding in the car, for example):

- Read to and with your child frequently, and check for comprehension by discussing what was read;
- Play rhyming games and similar games involving words with the same beginning sounds or same vowel sounds;
- Practice recognizing the memory words from Phase 4: the; I; to; and; do; me; he; go; no; you; and your;
- Practice the various vowel sounds for the six vowels (including "y");
- Practice addition and subtraction problems using the "call yourself" method;
- Practice addition and subtraction problems using the Zero Rules;
- Identify letters as vowels or consonants;
- Practice counting by 2's, 3's, 5's and, especially, 10's (this will help us in our base ten place sys-

20 Or possibly not.

tem work next chapter);

• Discuss the fact that the even numbers are those you say when you count by 2's and the odd numbers are all the rest of the "counting numbers" ("integers");

• Do fraction problems with a number of coins, blocks, etc. For example, take a group of twelve pennies and ask your child to divide it in half, etc.;

• Give your child lots of paper and many opportunities to write or draw;

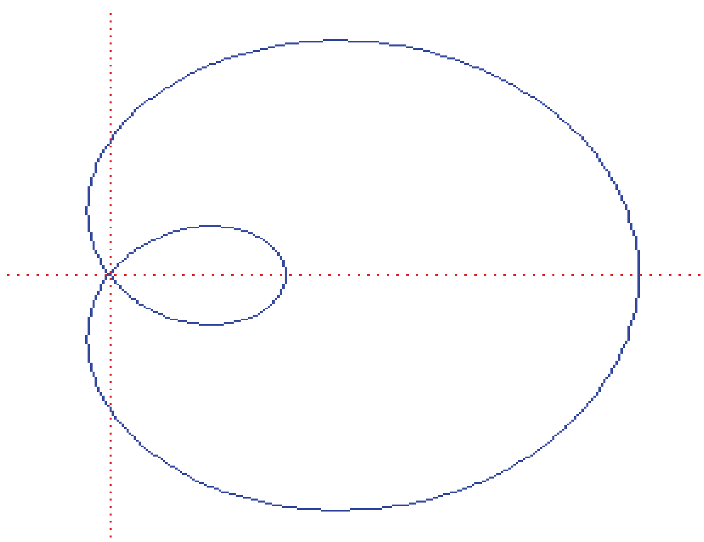
• Sing the "I Don't Know Any More" song (see Figure 11.8);

• Practice dividing shapes into fractions. Start with ½ -- draw some shapes (circles, squares, etc.) and have your child divide them in half. Draw a few more circles and divide them into two obviously unequal parts and let your child correct you – "Those are not halves, Daddy, they aren't the same." (Do what you can to elicit this correction – kids remember parental errors forever!);

• Say words to your child and ask her to write them;

• Play with various drawings of intersections, lemniscates, limaçons (see below)[21], and the like to discuss the ideas of interior, boundary, and simply connected;

Limaçon of Pascal



21 Picture taken from www-history.mcs.st-andrews.ac.uk/history/Curves/Limacon.html

- Discuss whether various things are two-dimensional ("2-D") – e.g., wall surfaces, sheets of paper, etc. -- or three-dimensional ("3-D") – e.g., balls, boxes, toys, etc.;

- Practice drawing boxes; and

- Discuss the idea of one-dimensional objects (a string, a line on a sheet of paper, etc.).

As we conclude Phase 4 and prepare for Phase 5, let us close with the words of Alfred Lord Tennyson:

> Here about the beach I wander'd, nourishing a youth sublime
> With the fairy tales of science, and the long result of Time . . . .
> For I dipt into the future, far as human eye could see,
> Saw the Vision of the world, and all the wonder that would be. . . .

Tennyson, Locksley Hall, 1842.

Chapter 13:
The Galileo Phase

Look at what we have accomplished in less than three years! Working at a leisurely pace -- on average, between two and three fifteen-minute sessions every week, plus occasional Field Work -- we have helped your child master the following, possibly before she is old enough to enter kindergarten:

- Recognition and production of all upper and lower case letters;
- Recognition and production of all numbers up to 99;
- Reading of paragraphs and stories at the early chapter book level;
- Addition, including addition of two-digit numbers;
- Subtraction;
- Recognition, production and geometric understanding of fractions; and
- An introductory understanding of key set theory concepts such as inclusion, intersection, boundary, etc.

And this is not all; in fact, this is only the tip of the iceberg, because we have also helped your child develop two very important attributes: a deep love of learning; and a strong and enduring sense of intellectual self-confidence. These attributes ensure that we have done more than just give your child a head start in the cognitive race; we have made her a faster and better learner. She will not taper off; her accelerated pace of learning will continue as long as she continues to experience a rich learning environment.

This chapter provides a good example of such accelerated learning. Over the course of the next eighteen months, your child will:

- Begin to read well enough to tackle regular chapter books;
- Master two-digit subtraction;
- Master multi-digit addition with carrying;
- Master multi-digit subtraction with borrowing;
- Multiply one and two digit numbers (with carrying);
- Work simple division problems; and
- Learn to add simple fractions.

If your child follows Sammy's path, she will be doing all of this – full-scale reading, advanced addition and subtraction, multiplication and division, and more -- before she enters first grade!

Does the Plan work well only with gifted children? Or is it the other way around – does participation in the Plan make it likely that the participant will qualify as a gifted child under standard elementary school criteria? To be honest, we do not know; our sample size is just too small to be able to answer these questions. Certainly, there is a strong correlation between full-scale Plan participants and children identified by their school systems as being gifted, but only time will tell in which direction the causal arrow points. Perhaps we have been fortunate in the children we have taught; perhaps our children have been fortunate to participate in the Plan . . . or perhaps there is a bit of truth in both propositions! In any event, I think you will agree that our results are extraordinary even for highly gifted children. And, with sixteen years of experience under our belts, we can say with some confidence that these results continue on into high school and college.

Are we going too fast? Are we subjecting children to hidden stresses by expecting so much of them? By now, you should know the answer to this: "No!" We are not "expecting" anything; we are merely getting out of the way and permitting learning to occur at the child's natural pace. It just so happens that a child's natural learning pace is extraordinarily fast! In any event, history teaches us that fast learning does not sate the appetite of the inquiring human mind, it whets it:

- In the space of about a year, Galileo saw for the first time: (i) mountains on the Moon; (ii) that the Milky Way is made up of stars; (iii) the four largest moons of Jupiter; and (iv) the phases of the planet Venus (like the phases of the Moon);

- Over a two-year period, Isaac Newton developed (i) differential and integral calculus, (ii) early versions of his three laws of motion, and (iii) the foundation of his famous inverse square law of gravity; and

- In one miraculous summer (1905), Albert Einstein published (i) a solution to the longstanding mystery of Brownian motion (giving birth to modern statistical mechanics), (ii) a solution to the longstanding mystery of the photoelectric effect (winning the Nobel prize and playing a key role in the quantum mechanics revolution), and (iii) his theory of special relativity (amazingly, he did not receive a Nobel prize for this or, even more shockingly, for his general theory of relativity, probably the single greatest known feat of human intelligence).[1]

Did Galileo, Newton and Einstein stop; did they say, "Well, that was fun, but the pressure of learning amazing new things is wearing me out; I think I will slow down, do a little gardening, maybe play a little canasta"? Of course not! They were having a GREAT time; they were having the times of their lives! These were miracle years, years of plenty, years that they would remember and treasure forever. They did not stop; they did not rest; instead, they continued to learn and to work with great passion, and they each achieved great things even after their miracle years.

"Did Galileo, Newton and Einstein stop; did they say, "Well, that was fun, but the pressure of learning amazing new things is wearing me out; I think I will slow down, do a little gardening, maybe play a little canasta"? Of course not!"

It is the same with your child. She is the intellectual heir of Galileo, Newton and Einstein; she is flexing her intellectual muscles and finding that she is STRONG! And it is exhilarating! She

1 Graduate students who study general relativity routinely become convinced that Einstein was a traveler from the future who brought the theory back with him. This is partly due to the difficulty of the theory and the complete lack of experimental motivation for it in the years in which it was developed, and partly due to the lack of sleep attending any career as a physics graduate student.

does not want to slow down; she wants to make this magic time last forever. Alas, this time cannot last forever, it will end some day. Peter Pan is not real; Puff the Magic Dragon silently fades away. Children grow up, and when they do they lose a good part of their extraordinary learning ability. So get rid of the choke chain; stop riding the brake. Let your child make the most of these magical years; let her spread her wings and fly.

Is your child going too fast? No, she is going just fast enough!

PART 1: Detailed Discussion of Phase 5: The Galileo Phase

Sample Homeworks: Sample Homeworks ##115 – 191.

Summary. With any luck, by the end of this Chapter – which could well occur before your child enters first grade – your child will be reading chapter books and doing math well above grade level.

Phase 5 Objectives.

- Continue to do Field Work regularly;
- Introduce three-digit and larger numbers;
- **Learn addition with carrying (regrouping);**
- **Learn simple two-digit subtraction;**
- **Learn subtraction with borrowing (regrouping);**
- **Read at a chapter book level;**
- Read chapter books for fun, solo;
- **Understand the base ten place system (ones column, tens column, hundreds column);**
- **Learn to read and identify fractions;**
- Add fractions having the same denominator;
- **Multiply;**
- **Divide;**
- **Multiply two-digit numbers;**
- **Multiply two-digit numbers with carrying;**
- **Work on curing Sammy's problems in pronouncing the "r" sound;**
- Introduce three-dimensional shapes;
- Continue and expand work on intersection pictures; and
- Introduce plane geometry (lines, rays, line segments).

Time Frame for Phase 5. Sammy took approximately eighteen months to finish Phase 5, starting at age 55 months and finishing at age 73 months, just about the age of a beginning first grade student. Again, homeworks were often done multiple times, with the result that there were approximately one hundred fifty (150) homework sessions over the eighteen months of Phase 5. This works out to about two sessions per week.[2]

Detailed Suggestions and Comments for Phase 5:

1. A Few Reminders.

• *The Outpost Approach*. Remember, we move fast, and we establish outposts first, then consolidate. Our children will be multiplying 23 x 45 well before they have memorized their simple addition tables. That is OK! In fact, that is more than OK; that is exactly as it should be.

• *Concrete Procedure Before Theory*. Although we are firm believers in exposing a young child to complicated ideas, we prefer to do so after they have established a "comfort zone" with the material. Thus, we generally begin any new area (e.g., addition with carrying, subtraction with borrowing, multiplication, etc.) by helping the child learn a "recipe" – a concrete procedure -- for handling the new type of problem. Once the child feels a sense of mastery over the new problem type, and thus develops a sense of confidence that will allow her intuitive learning apparatus to operate unencumbered by doubt or fear, we feel free to throw interesting theoretical ideas into the mix – but always in a concrete way! One might say that our children are able to build elaborate structures of knowledge because they begin with concrete foundations. You will see what this means through many examples.

". . . our children are able to build elaborate structures of knowledge because they begin with concrete foundations."

• *Introductory Discussions Should Be Repeated.* Throughout this book, we provide fairly detailed "scripts" of sample discussions between you and your child, usually in the context of an important new idea. Of course, these "scripts" are intended to serve as resources rather than requirements: you should feel free to depart from them in any way you wish. But, please note that use of these scripts does not reduce the need for repetition! Young children learn in large part by repeated exposure to new material; it is this process of repetition that allows the formation of connections between old knowledge and new, the hallmark of inductive learning. We have noticed over the years that parents who rely on our "canned scripts" tend to use them only once and then move on. It is far better to use these scripts multiple times, or (even better) to cover the same introductory material in slightly different ways, with your own "scripts," multiple times. The point here is that repetition is especially important with respect to introductory material – the very type of material that

2 The truth is beginning to emerge; despite our admonition to do four homework sessions per week, in practice we never actually achieved that! We do recommend four sessions per week, but Field Work remains very important even in this advanced Phase, and outside reading by the child is perhaps the most important component of all at this point.

is most often covered by our canned scripts.

2. *Sample Homework #115.* Notice the high level of reading that we routinely expect at this point. Isn't this a nice little story? Discuss it with your child. What does it mean? Is the Mom a duck? If so, what does that mean about the little girl? Or is the Mom just being funny? There is no right or wrong interpretation; it is just an opportunity to have a fun discussion.

3. *Sample Homework #116*. We introduced our first three-digit number, "100," in Sample Homeworks #111 and 112. Having established an outpost, we are now ready to get much more serious about three digit numbers. And, in the process, we will find that we have launched a major new project: understanding our very interesting base ten place system.

- *Is the Base Ten Place System Too Complicated For Our Fragile Young Learners?* Now that we are beginning to work on three-digit numbers, we have an opportunity to introduce the columns we use in our base ten numbering system – in the case of three-digit numbers, the ones column, tens column and hundreds column. But this is very abstract and complicated. Should we defer approaching it until the child is older?

  Let me give a delicate and nuanced response: No! (You knew that, right?) Our numbering system is indeed complicated, and we would be beating our heads against a stone wall if we tried to teach it at a theoretical level to a young child. But that is not our way, grasshopper. We will not try to "teach" it – but we will discuss it, and float it out into the general atmosphere over a long period of time, and allow the child's inductive learning apparatus to work on it. In other words, we will approach it in a gentle way over a number of homework sessions. And, we will do so using a familiar technique: we will approach the abstract idea of a base-ten place system very concretely, giving your child a solid foundation that will enable subsequent inductive expansion of her understanding.

A Brief (Optional) Digression: An Overview of the Base Ten Place System For <u>Parents</u>. Adults think and learn differently than children. Although we are about to begin a long and gentle process of presenting the important concepts of our base ten place system (also known as our "number system") to your child in our standard inductive fashion – i.e., many concrete examples, questions without definite answers, repeated exposure, etc. -- some parents prefer in this difficult abstract area to have a clear understanding at the outset of our targets and goals. What do we want the children to learn about the base ten place system? In fact, what exactly is the base ten place system?

This overview section attempts to answer these types of questions by providing parents with a brief, carefully organized, step-by-step discussion of the base ten place system. Of course, we would never use this kind of top-down, packaged presentation with our young Plan participants; we want them to extract the knowledge themselves from repeated exposure to a carefully prepared environment rich in relevant content. We understand that by allowing our children to learn, rather than by teaching them, they gain a much deeper, more intuitive understanding of the ideas – they become smart rather than merely knowledgeable. But adults are not as smart as children; with parents, we will settle for merely knowledgeable!

This brief section is intended to give you a sense of our goals with respect to the base ten place system and a better understanding of our approach to this important topic over the next twenty or thirty Sample Homeworks. This section is not required reading on your part! The Sample Homeworks are designed to give your child a sufficient level of exposure to concrete instances of these general abstract rules even if you do not understand the rules yourself. In fact, we should emphasize that the concepts discussed in the bullets below are abstract and should not be "taught" to your child! Instead, the Sample Homeworks and the related detailed suggestions provide you with many opportunities to discuss concrete instances of these general ideas with your child. We can trust your child to build a basic, intuitive understanding of the underlying rules from these repeated concrete exposures.

Here is the parents' summary of our base ten number system:

- *Ten Symbols.* We write all of our numbers using only ten symbols (hence the label "base ten"), the numbers "0" through "9." All larger numbers are written using a combination of these ten symbols. For example, although we could use a new symbol for the number twelve – maybe we could say that the symbol for "twelve" is "Ŋ" —we do not use a new symbol. Instead, we write "twelve" using a "1" and a "2" from our basic list of ten symbols ("0" through "9").

- *Syntax of a Number: Order and Column.* If we are going to use only ten symbols to write every number, we must use something else beside the symbols to distinguish the numbers. And, of course, we use the placement of the symbols – the order of the numbers and the columns in which they appear. "12" is a different number than "21," and both are different than "120." Why? Because a number is defined not only by the symbols that appear in it, but also by their order and their placement in particular columns. The rules of order and placement that help define a number constitute the "syntax" of our numbering system – an exciting thought, because syntax is a language concept, which suggests that our young children can learn a lot about it through their innate language learning ability.

- *Column Syntax Requires Zeroes.* If the order of symbols in a number were the only syntactic component of a number, we would not need to worry about columns, and zero would be a much less important number. But in fact a "2" in the hundreds column and a "1" in the tens column (210) is a very different number than the number having the "2" and the "1" in the same order but in the tens and ones column, respectively (21). So the column in which a number is placed is critical, and this means that zeroes are critical as well. For example, I might write the number "two hundred" as "2," understanding in my own mind that the "2" is in the hundreds column. But a reader would never know that I intended to put the "2" in the hundreds column! We need to use zeroes to "hold the places" in the ones and tens columns in order to make our intention clear: "200."

o *Columns Are Powers of Ten*. In base ten, each column is ten times the column to the right of it – so the columns go in powers of ten: 1's; 10's; 100's; etc.[3] Why? Because when we get to the number "9" in any column, we have used up all of our ten symbols; there is nothing else to write in that column. So we have to move over to the next column.

o *Why Base Ten?* Why base ten instead of base twelve or base six? Count your fingers! But other bases can be useful: for example, my father recently retired at the age of forty-one (he uses base sixteen).

• *Back to Business: The Number "100," and the Importance of Place and Order*. Now we are ready to return to our discussion of Sample Homework #116. After reading the initial row of letters and the first few numbers, your child will come to the number "100," which of course she has seen before in earlier Sample Homeworks. This marks the beginning of her work on the base ten place system! You may wish to stop and have a brief ceremony![4] Or you can plow right ahead and discuss the number "100," perhaps in a manner similar to the following (remember, we do not believe in rhetorical questions; every time you ask your child a question in these homework sessions, you should pause and look at her and pay attention to her answer):

> **" . . . we do not believe in rhetorical questions; every time you ask your child a question in these homework sessions, you should pause and look at her and pay attention to her answer . . . "**

3 Note that 10^0 is defined to equal one.

4 I like to do a small ceremony in honor of our base number, 10, in which the parent slowly counts from one to ten, with the child saying "Q" after each number. Thus, the dialogue goes like this:

Parent: "One …."

Child: "Q."

Parent: "Two …."

Child: "Q."

and so on. At the end, after you say "Ten" and your child responds "Q," you should say "You're welcome!" (Get it? "Ten-Q, you're welcome.") And then you and she should eat corn-dogs in honor of such a lame, corny joke.

Sample Transcript: 100 and the Place System

"Do you see this number, a '1' with two zeroes after it? [Give her a chance to answer.] Do you know what this number is? [Give her a chance to answer.] This is a very fun number, a very big number. It is ONE HUNDRED! Can you say ONE HUNDRED? [Give her a chance . . . . OK, you get the idea.]

Very good! One hundred! How many zeroes are there in the number 100? Yes, that's right – two zeroes. Are they in front of the '1' or are they behind the '1?' That's right, they are behind the '1' – they are to the right of the '1.' [5]

[Write down the number '001.'] Is this number one hundred? No, of course not! It has two zeroes, but are they in the right place? No, they are in the wrong place! They are in front of the one, on the left side of the one. That does not work – that is not one hundred. Can you write the number '100' the correct way? Can you put the two zeroes in the right place? Very good!

Let's look at the number '001' again. Do you know what number this is, really? It is just the number '1.' Do those zeroes matter at all? NO, they do not! Zeroes in front of the number do not matter. It is only other zeroes that matter.[6]

Do we need the zeroes to write the number 100? Well, let's try it without the zeroes. [Write a '1.'] Here, we left out the zeroes. Is this number 100? NO, of COURSE not! What number is it? That's right – it is the number 'one.' It is CERTAINLY not the number '100!'

Well, maybe we can just leave off one zero and still write '100.' Do you think that will work? NO – you are probably right, but let's try it. [Write '10.'] There, we only used one zero. Is that number one hundred? No, of COURSE not – it is ten! So what do you think? Do we need both zeroes? Yes, that's right, we do.

But, isn't that kind of funny? After all, how much is zero? Is it a lot?

5 If your child has not mastered left versus right, you should add this to the Field Work.

6 Actually, this is not a correct statement when decimals are considered. A more precise statement would be that, "zeroes occurring between any non-zero digit and the decimal point – on either side of the decimal point – are needed; all others are superfluous." For now, as we prime your child's inductive pump, we are content to live with this small inaccuracy.

> *No, of COURSE not. It is zero; it is nothing at all. So why do we need zeroes to write a hundred? Why do we need zeroes, when they are just nothing? What do you think? [Give your child some time to wrestle with this and let her try to explain it.]*
>
> *Why do we need the zeroes? Well, I guess the thing is, you need the zeroes to put the '1' in the right place, don't you? [Discuss this idea.]"*

• *Summary of Discussion of "100."* In our peculiar, long-winded way, we covered the following important ideas in the above discussion of the number "100":

- o We need zeroes to write the number one hundred;
- o In fact, we need exactly two zeroes; and
- o The zeroes and one have to be in the right <u>order</u> and in the right <u>places</u>.

We will build on these initial place-system ideas in the following Sample Homeworks.

• *Reading the Number "101."* At the end of the row of numbers, we encounter another three-digit number, "101." First help your child read this number, then have a discussion similar to that described above for the number "100." Here, you should concentrate on the ideas that you need a zero for this number (i.e., "11" is not the same as "101") and that the place and order of the numbers is critical (compare "101" with "110" and "011"). You might want to discuss why the "0" in "011" is not necessary.

4. *Sample Homework #117*. We continue our gradual introduction of the base ten place system.

• *The Ones and Tens Columns: the Difference Between "21" and "12."* Note that the second row consists of the numbers '21' and '12' next to each other. We want to use these two numbers to help your child understand the importance of "place" in our number system, and to introduce the ones and tens columns. You might discuss this in a fashion similar to the following:

Sample Transcript: 21, 12 and the Place System

> *"[Point to the '21'] What is this number? VERY GOOD! Yes, it is 'twenty-one.' Does it matter, when we write a number like 'twenty-one,' where we put the numbers? Is it OK to write twenty-one like this [point to the '12']? Can we put the '1' and the '2' anywhere we want and still have 'twenty-one'? Of COURSE not – we have to write it like this: [point to the '21']. We have to write the two first and then write the one. If we write it in the other order, like this [point to the '12'], what do we get? YES, we get twelve. Is that the same as twenty-one? Of COURSE not!*

> *It matters where we put numbers – in what place we put them, doesn't it? After all, '12' and '21' are not the same, are they?*
> *Because it is important what place numbers are in, we give the places names. [Write '15' in the space to right of '12' on the second line.] What number is this? GOOD! Fifteen! That's right. Now, the first place, the place the five is in – that is called the 'ones column.' What is it called? Right, the 'ones column.' In the number fifteen, what number is in the ones column [point to the '5']? Right! The five is in the ones column. And what number is in the TENS column [point to the '1']? Right, the one is in the tens column. The first column is the ones column; the second column is the tens column. Let's do a few more. [Repeat, with discussion, for '16,' '23,' '56,' and '90' -- you will have to write each of these numbers yourself in the blank space on the second line.]"*

• *The Hundreds Column.* The third line begins with '100.' Here is our opportunity to introduce the hundreds column. You might, for example, proceed as follows:

Sample Transcript: The Hundreds Column

> *"Now, we have an even bigger number [point to the '100' at the beginning of the third line], we have one more column. What number is this? Yes, it is 'one hundred.' Let's talk about it the same way we have been talking about these numbers [point to the two digit numbers in the second row[. Here, what number is in the ones column? [Point to it.] Right, there is a zero in the ones column! And what number is in the tens column? [Again, point to it.] Right again! There is a zero in the tens column! And what number is in the hundreds column? [Point to the '1'.] Right – for three-digit numbers, the new column is the 'hundreds' column, and if we put a one there, we get one hundred – get it? A ONE in the HUNDREDS column means 'one . . . hundred.'"*

• *Practice Place System Naming With the Rest of the Numbers In Row Three.* After discussing "100" in a place-system kind of way, as described above, do the same thing (in a more abbreviated fashion) with the other numbers in the third row of the Sample Homework. For example, as you look at "23," ask your child, "What number is in the tens column?" ["2"] Then point to the "3" and say, "and the '3' must be in the . . . ." [The proper answer is, of course, the "ones column."] Note that we are encouraging both recognition – where the child must <u>recognize</u> a named column -- and production – where the child must herself <u>produce</u> the name of the column.

When you get to the last number, "110," you may wish to review the importance of the zero, and of the placement of the zero – just as in Sample Homework #116 above, but this time using the names of the columns in your discussion. (E.g., you might ask, writing "101" as you speak, "Would this number be the same if we put the zero in the tens column and the ones in the hundreds and ones columns, like this?")

• *Shapes and Fractions.* Ask your child to divide the left-hand shapes into halves and the right-hand shapes into thirds. (Thirds is much harder, especially for the triangle! You might want to ask your child why it is so hard to divide the triangle into thirds.[7]) Remember that the pieces of any divided shape must be equal in size, but not necessarily in shape.

5. ***Sample Homework #118.*** In this important Sample Homework, which probably should be repeated several times, we do four things (not necessarily all in the same session): (i) learn a trick for reading three-digit numbers; (ii) continue to work on the ones, tens and hundreds columns; (iii) learn how to pronounce words ending in "y;" and (iv) perhaps most importantly, introduce a strategy for reading compound words (in this case, "firemen").

• *How to Read a Three Digit Number.* As you tackle the four three-digit numbers in the second row of this Sample Homework, you should point out to your child that one easy way to read three digit numbers is to do the following three steps: (i) read the first (left-most) digit; (ii) then say the word "hundred;" and (iii) then read the last two digits as a separate number. (At first, your child may wish to cover up the number in the hundreds column with her finger after she reads it, making it easier to read the remaining two-digit number. If used, this technique should be discontinued after a few days and replaced by a "mental" covering up of the number in the hundreds column.) So, for example, we read "210" by (i) reading the first digit, "two," then (ii) saying "hundred," then (iii) reading the last two-digit number "ten." If you use this method, you may want to point out that a number like "04" is just read as "four."

• *Place System Naming Questions.* Continue to emphasize the place system in discussing two- and three-digit numbers, both in the row of numbers and in the addition and subtraction problems. At this point our primary focus continues to be on teaching the names of the three relevant columns: the ones column; the tens column; and the hundreds column. We do this by asking the child several questions about each number:

> *"What is this number?"*
> *"What number is in the ones column of this number?"*
> *"What number is in the tens column of this number?"*

7 There is no need to reach a conclusion on this point; we are just promoting mental activity! Here is a hint if you are interested: It would also be hard to divide the triangle in half if it were not an isosceles triangle (having two equal sides). Does it help when the shape has a symmetry that allows each fractional piece to be not only the same size, but also the same shape?

"What number is in the hundreds column of this number?"

(For a two-digit number, the answer to the last question is, of course, zero or no number.)

Alternatively, we can switch gears and ask the same questions from a different perspective (production rather than recognition of the columns). For example, for the number "123," we might ask:

> *"What is this number?"*
> *"The '2' is in which column?"*
> *"The '1' is in which column?"*
> *"The '3' is in which column?"*

Or, of course, we can mix both types of question:

> *"What is this number?"*
> *"The '2' is in which column?"*
> *"What number is in the ones column of this number?"*
> *"The '1' is in which column?"*

• *Words Ending in "y."* As you read the words ending in "y" (sky, try, etc.), mention words such as "Sammy" that end in a "y" having a different vowel sound. Explore the different sounds that the letter "y" can have. When is it a consonant (as in, e.g., "yellow")? When is it a vowel? Recall our vowel chant ("and sometimes y")!

• *Compound Words.* Notice the word "firemen." Here the "Silent e" rule applies to an "e" in the middle of a word, instead of at the end of the word. Ask your child why; see if she can figure out that this is a compound word, a word made up out of two smaller words. How do we read such a word? We use . . . .

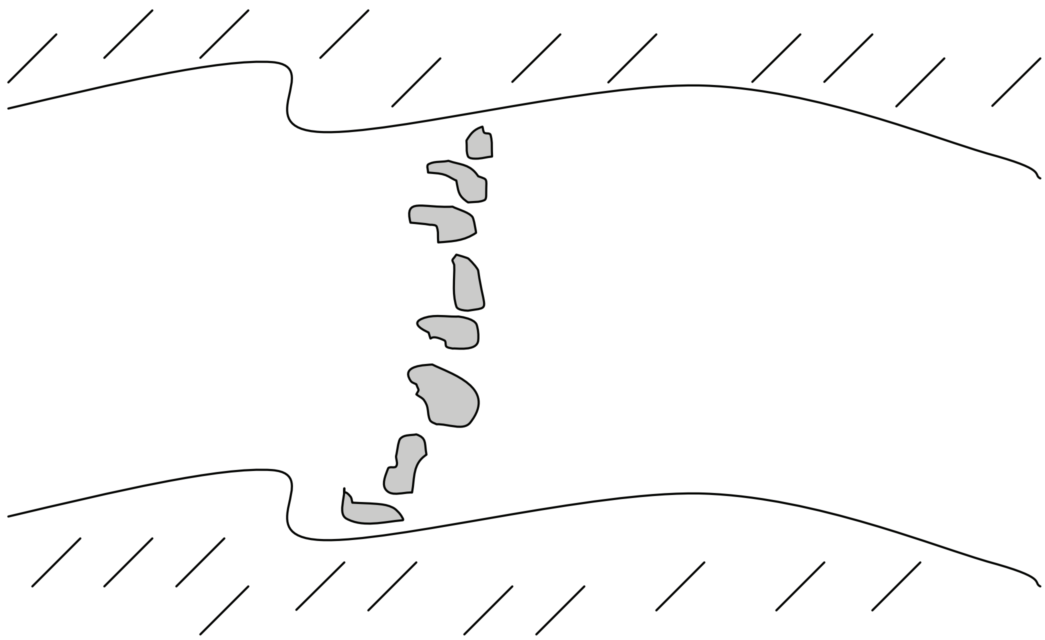
o *The Stepping Stone Approach.* The key to reading any multi-syllable word is to use patience and take it in steps, sounding it out in chunks. The word may look daunting at first, but broken into little pieces, it becomes easy.

You might want to discuss this approach with your child as follows:

Sample Transcript: The Stepping Stone Approach

> *"Sometimes we have to work on hard things in little steps, instead of trying to do the hard thing all at once. Here is an example. Suppose we just got brand new shoes, and we are out walking in the woods when we come to a shallow creek. It is too wide to jump across; if we tried we would get our*

brand new shoes wet. And although that might be fun, Mom would be very upset! [8] But we really want to get to the other side. It is too wide to jump, and we feel like giving up. After all, what else can we do? It is just a creek, full of water, with a lot of big flat stones sticking out of it. Just a bunch of big flat stones sitting there, nice and dry. We are really stuck, aren't we? [Hopefully, your child will suggest walking across on the stones. If necessary, feel free to draw a quick sketch like the one shown below.] Of course, there is an idea! We do not have to get across all at once! It will work just fine if we take our time and take a few extra steps to get across. We can just step from stone to stone and – presto – there we are on the other side!



Reading a hard word is like that. It would be hard to read the whole word, so we just start by reading a little part of it. That is like stepping out onto the first stone. Then we read another little part, and another, and pretty soon, we have read the whole word. So, let's try that approach with this word [point to "firemen"].

Just for now, let's start by ignoring the last part of this big word. Let's cover up the end of the word. [Use a finger to cover up the letters "emen" at the end of the word. NOTE that we are covering up the "e" at the end of "fire"

8 We realized early on that we had to add this "upset Mom" line to the story, because children were not seeing the downside of getting their shoes wet!

– we are doing this for a reason!]

Let's just take this word a piece at a time – one stepping stone at a time – and let's just read the first part of the word. [Have your child sound out "fir."] Very good! "Fir"! [Slide your finger over enough to reveal the "e."] Whoops! Wait a minute

-- we have an "e" here at the end of what we were sounding out! It sure looks like a silent "e," doesn't it?

Let's go ahead and read the word with a silent "e" now. [Your child should read the word "fire."] Very good! Now, let's just remember that our big word starts with fire, and let's read the second half of the word. [Slide your finger over to cover "fire" and reveal "men."] Can you read the second part? ["Men."] Good!

Now let's put the two parts together into one word. Say the first word then move right into saying the second word – say them together, one right after the other. ["fire – men."] Good!

And that is how we can read a word like 'firemen.' We read the first part – 'fire' – then we remember that part and go ahead and read the second part – 'men' – and then we put them together: 'fire…men.'" And we don't really have to cover letters up with our fingers – we just did that since it was the first time."

This "Stepping Stone Approach" is of great general value. For example, you can often solve a hard math problem by just taking a step, by doing something, anything, that gets you started. We will certainly find many occasions to refer back to this "Stepping Stone" approach!

" . . . you can often solve a hard math problem by just taking a step, by doing something, anything, that gets you started."

6. Sample Homework #119.

• *Base Ten Place System Work*. Continue the base ten place system work begun in the preceding Sample Homeworks -- remember to use the trick for reading three-digit numbers (see discussion of Sample Homework #118, above) with "200."

• *Reading From a Book!* Now we take a major step forward: we ask your child to read aloud to you from a book! Here is a list of some of our favorites:

"E" = EASY, "M" = MEDIUM, "H" = HARD

Calvin and Hobbes books, Bill Watterson (M)
Where the Wild Things Are, Maurice Sendak (E)
When I Have a Little Girl, Charlotte Zolotow (M)
Goodnight Moon, Margaret Wise Brown (E)
Charlotte's Web, E. B. White (H)
Giving Tree, Shel Silverstein (H)
Blueberries For Sal, Robert McCloskey (M)
The Story About Ping, M. Flack and K. Wiese (M)
The Very Hungry Caterpillar, Eric Carle (E)
Are You My Mother?, P. D. Eastman (E)
One Riddle, One Answer, L. Thompson and L. Wingerter
See the Circus, H. A. Rey (E)
Horton Hears a Who, Dr. Seuss (M)
The Big Orange Splot, Daniel Manus Pinkwater (M)
Much Bigger Than Martin, Steven Kellogg (M)
Make Way For Ducklings, Robert McCloskey (M)
George and Martha books, James Marshall (M)
Little Bear books, Else Minarik (M)
The Grouchy Ladybug, Eric Carle (E)
The Hat, Jan Brett (M)
Mike Mulligan and the Steam Shovel, Virginia Burton (M)
The Year at Maple Hill Farm, Alice and Martin Provensen (M)
Brown Bear, Brown Bear, What Do You See?, Bill Martin, Jr. (E)
Horton Hatches the Egg, Dr. Seuss (M)
The Treasure, Uri Shulevitz (M)
The Snowy Day, Ezra Keats (M)
Once a Mouse . . ., Marcia Brown (M)
How the Grinch Stole Christmas, Dr. Seuss (M)
Miss Nelson Is Missing, Harry G. Allard (M)
Caps For Sale, Esphyr Slobodkina (M)
The Napping House, Audrey Wood (M)
There Was An Old Lady Who Swallowed a Fly, Pam Adams (E)
Alexander and the Terrible, Horrible, No Good, Very Bad Day, Judith Viorst (M)
Bedtime for Frances, Russell Hoban (M)
Where The Sidewalk Ends, Shel Silverstein (H)
Six Creepy Sheep, J. Enderle and S. Tessler (M)
Boxcar Children # 1- 19 , Gertrude Chandler Warner (H)

If You Give a Mouse a Cookie, Laura Joffe Numeroff (M)
Madeline books, Ludwig Bemelmans (M)
Angelina and the Princess, Katharine Holabird (M)
Curious George books, H. A. Rey (M)
Berenstain Bears books, Stan and Jan Berenstain (H)
Little Critters books, Mercer Mayer (M)
Bailey School Kids books (Daden and Jones) (H)

It is no accident that the Sample Homework sentence, to be read following the book reading, is "I am a big girl." [9]

• *Shapes.* Ask your child to divide each shape in half (discuss how this applies to the parabola). Then have her show you the boundary, interior and exterior of each shape – use this as an opportunity to discuss the differences between the parabola and the other shapes. (The main difference is that the parabola is not a "closed" shape and thus does not have an interior or exterior. Also, because it is not closed, the parabola cannot serve as the boundary of a two-dimensional shape, unlike a circle, for example.)

7. *Sample Homework #120.* Another great little story, based on an oral description by Sammy. (Children love it when you show that you are listening to them!)

8. *Sample Homework #121.* Now that your child is at least somewhat familiar with the names of the columns, we can begin to think about what the various columns actually mean. To that end, after reading the letters and sentence in the first two lines of the Sample Homework, we expand our discussion of the row of numbers in the third line as follows, number by number (starting with the first number, 34):

• *Continue Place System Work From Prior Sessions.* Begin by asking your child some of the questions about the various columns described in "Place System Naming Questions" in Sample Homework #118. (These questions should be a part of most sessions for quite a while.) For example, for the number 34, we might ask:

"What is this number?" (thirty-four)
"The '3' is in which column?" (tens column)
"What number is in the ones column?" (four)

• *Place System: How Many Tens?* Then, before moving on to the next number, we add a new wrinkle, asking, "And how many tens are in this number?" Our discussion of this question

9 We almost hesitate to point out that if your child is named Fred or John, for example, you should alter this sentence slightly so as to avoid gender confusion.

will be slightly different depending upon whether we are dealing with a two-digit number or a three-digit number.

o *How Many Tens (Two-Digit Number)?* If we are dealing with a two-digit number such as 34, we let the child stew for a while, then point out that the number in the tens column tells us the answer. For example, there are three tens in "34" because there is a "three" in the tens column. And, the number in the ones column ("4") is the number "left over" – i.e., the number that is not in a group of ten.

- You should use pennies to illustrate this until your child is comfortable with the idea. For example, if the number is 34, start with thirty-four pennies and count out groups of ten pennies, showing that there are three groups of ten pennies (of course, as with all concrete exercises, your child should be manipulating the objects, in this case pennies.). Since there are three groups of ten pennies, there are three tens in 34 – and three is the number in the tens column. So it all works! Now have your child replace each group of ten pennies by a dime – just another way of differentiating one "ten" from one "one!"

- Also, there are four pennies left over, and that this is the number in the ones column for 34.

o *How Many Tens (Three-Digit Number)?* The same questions – How many tens are there in this number? How many ones are left over? -- can (and should!) be extended to three-digit numbers, but the answer to the first question (how many tens are in the number?) must be determined a little differently than in the case of a two-digit number. With a three-digit number, the number of tens cannot simply be read off from the tens column; instead, the child should ignore the ones column and read the remaining two-digit number – that will be the answer. For example, if the question is, "How many tens are in 364?", the child should ignore the "4" in the ones column and read off the remaining two digit number from the hundreds and tens column: "36." That is, there are thirty-six tens in 364. At first, the child may wish to cover the digit in the ones column with her finger, to help her see the remaining two-digit number. (You may wish to point out that this method works just fine for two-digit numbers as well!) Again, a few concrete exercises with pennies would be a good way to illustrate this idea. [10]

- *Two Additional Standard Place System Questions.* Henceforth, we may feel

10 Over the years, we have learned that almost nothing actually "goes without saying!" Thus, please do not be offended by our recommending the obvious -- that you use only low three-digit numbers for these concrete counting exercises. For example, it takes a very long time to count out 983 pennies!

free to build on the lessons of this Sample Homework by adding the following questions to our standard list of questions about two- and three-digit numbers:

> *"How many tens are in this number?;" and*
> *"How many ones are left over?"*

9. ***Sample Homework #122.*** Guess which word in the story was Sammy's favorite!

10. ***Sample Homework #123.*** Like Sample Homework #118, this is an important Sample Homework, one that might well be repeated several times.

• *A Reminder.* As always, do the usual place system work with selected two-digit numbers and with each three-digit number in the rows of numbers. (This should also be done on occasion with the numbers that are being added or subtracted.) As a reminder, the "usual work" boils down to asking one or more of the following types of questions (e.g., for the number "104"):

> *"What is this number?" [one hundred four]*
> *"The '1' is in which column?" [hundreds column]*
> *"What number is in the ones column?" [four]*
> *"The "0" is in which column?" [tens]*
> *"How many tens are in this number?" [ten]*
> *"How many ones are left over?" [four]*

• *Base Ten Place System: Two New Questions.* Now may be a good time to introduce two additional questions about the two- and three-digit numbers. It is best to begin with a relatively small two-digit number (e.g., 34), simply for logistical purposes.

o *How Many Ones?* First, after asking "How many ones are left over?," you might say, "Now, let's think about something a little different. We have been talking about how many ones are left over. Now let me ask you a different question: How many ones are in this number? Not just how many are left over, but how many ones are in this number in all?"

To approach this, have your child count out the number (e.g., 34) using pennies. Counting by ones, she will see that there are 34 ones in 34. With luck, she will quickly see the connection; one just reads the number to see how many ones are in it.

Your child may have difficulty with this; that is OK; it is our norm. If she is having trouble arriving at this rule, give her some gentle help.

Remember that we create the desire to learn by presenting the child with problems that cannot easily be solved without new ideas. Because the new ideas are useful, your child (who is a member of the most utilitarian of all groups – young children!) will remember them well.

o *Tens and Ones.* The second new question about place value that we will introduce is to ask the child to describe a two-digit number, "34" for example, as being "three tens and four ones." Again, we occasionally use pennies to reinforce this. (Feel free to use dimes to represent groups of ten.) This can be extended to three digit numbers as soon as your child gets the hang of describing two-digit numbers in terms of "tens" and "ones" -- e.g., "467" can be described as "four hundreds, six tens, and seven ones."

• *Reading "igh"Words*. Focus on the "igh" sounds contained in each of the words in the word row. Ask your child to sound them these letters out – of course she will have a tough time of it, but that is OK. Allow her to struggle a bit, then tell her that "igh" sounds like "eye," so that "ight" sounds like "ite," and then laugh with her about what a crazy language we have! Note that the word "might" also occurs in the sentence near the end of the Sample Homework.

" . . . laugh with her about what a crazy language we have!"

• *Fraction Work.* Here we build on the work begun in Sample Homework #71. You may wish to revisit our discussion of that Sample Homework to refresh your memory!

o *Read the Fractions.* Have your child read each fraction, and ask her to identify the numerator and denominator. This should be done part of the time as a recognition task ("What is the numerator of this fraction?") and part of the time as a production task ("What part of the fraction is the "6?").

o *Meaning of Denominator*. As your child reads each of the first four fractions (1/2, 1/3, 1/4, 1/5), ask her how many of that fraction there are in one whole thing (a whole pie, a whole circle, etc.).

• For example, after your child reads "one half," you might say, "Very good! Now, how many halves are there in one? How many halves of a pie are there in one whole pie?"

• Draw a circle and divide it into two (equal) halves to help.

• Point out that the answer -- two -- is just the denominator of the fraction. Ask your child what part of the fraction tells you into how many pieces the circle should be divided? [denominator – 2]

• Ask your child to color one half of the circle that you drew. What part of the fraction tells us how many pieces to color? [numerator -- 1]

• Repeat this process for the next three fractions: 1/3; 1/4; and 1/5.

o *Meaning of Numerator*. When your child gets to the last fraction, "2/5":

• Ask her again how many fifths there are in one whole circle or pie.

• Ask her where she can look in the fraction to get the answer (the denominator).

• Then, ask her to read the numerator, "two."

• Say, "Here we are not talking about one fifth, are we? Instead we are talking about . . . . that's right, TWO fifths, because the numerator is two."

• Draw a fresh circle. Ask your child to divide it into fifths. Help her as necessary (this is hard!).

• Ask, "Could you please color two fifths of the circle?"

• Discuss the key ideas that (i) the denominator tells you how to divide the circle – the total number of pieces of pie – and (ii) the numerator tells you how many of those pieces you get.

"(i) the denominator tells you how to divide the circle – the total number of pieces of pie – and (ii) the numerator tells you how many of those pieces you get."

• Practice with other fractions suggested by your child.

11. ***Sample Homework #124.*** Note that at the top of the Sample Homework we now ask the child to write a word of her own choice.

• *Thinking About Two-Digit Addition and Subtraction Problems In Terms of the Base Ten Place System*. The groundwork has been laid. Now that your child has been introduced to the base ten place system, two-digit addition and subtraction problems can be discussed in terms of the ones and tens columns! Ask your child to describe how she will tackle these problems – see if you can get her to describe the process as first adding (or subtracting) the numbers in the ones column and then adding (or subtracting) the numbers in the tens column. If she does not use these ideas in her description, you should gently introduce them. Help as needed -- this is important for later work in carrying and borrowing. This approach to two-digit addition and subtraction problems should be used in ensuing homework sessions as well.

• *A Note About Sample Homeworks That Are Not Discussed*. You will note that many Sample Homeworks are simply not discussed in this "Detailed Discussion" section. (For example, following this discussion of Sample Homework #124, we will skip Sample Homework #125 and proceed directly to a detailed discussion of Sample Homework #126.) This does not mean that the omitted Sample Homeworks are unimportant. Au contraire![11] Such Sample Homeworks allow consolidation of new knowledge through repetition, one of the most important elements in our inductive learning approach. They are not discussed in detail simply to avoid presenting you, the adult reader, with repetitive material. You should of course feel free to refer back to discussions of earlier Sample Homeworks as appropriate to help you conduct homework sessions using these "consolidation" Sample Homeworks.

12. ***Sample Homework #126***. Here we approach the place system from three perspectives.

• *The Usual Place System Routine*. First, go through the usual place system routine with the row of numbers at the top.

• *Ten Symbols Only (In Base Ten)*. Second, when your child gets to the row of numbers in the middle of the Sample Homework, note that the numbers are listed in increasing order from zero to nine, followed by the Greek letters alpha and beta. For fun, suggest to your child that alpha stands for "10" and beta stands for "11." Let your child point out that this is not true. Discuss how the place system means that we do not need separate symbols for "10" and "11." (Instead of needing a new symbol, we use old symbols and pay attention to where we put them – i.e., to their places.)

• *Place System Tables*. Finally, for fun, play with the three place-system tables located in the bottom half of the page. For example, you could say a two or three digit number to your child and have her write it in the tables, placing each digit in the correct column.

o If you say "twenty-three," your child should write a "2" in the tens column and a "3" in the ones column. It would be best if she used the left table or the center table, each of which has only two columns (ones and tens), but it is OK if she uses the right-hand table (which has three columns).

o In fact, there is a learning opportunity if your child uses the right-most table for a two-digit number! In that case, ask her what goes in the hundreds column. Of course, the correct answer is nothing, but then why can we leave it blank? Why don't we have to write a zero there? After all, when we write "100," we cannot just leave blanks in the ones and tens columns! Discuss with your child – why

11 Occasionally, in moments of strong emotion or when making an important point, we inadvertently lapse into French.

don't we write zeroes out to the left of our numbers – why don't we write "23" as "023" or "0023", etc.? It is OK if you don't come up with a satisfactory answer in this session – the discussion is more important than the answer.[12] (Sometimes we suggest an answer such as, "We only need zeroes to hold places on the 'inside' of the number, not on the 'outside' of the number.")

13. *Sample Homework #130.* In this Sample Homework and the next, we introduce a wide variety of "oo" words. Point out the two basic sounds of "oo" (as in "book" and "zoom").

14. *Sample Homework #131.*

• *"Roof."* You may want to discuss the fact that the word "roof," which occurs in the Sample Homework's sentence, is really two different words, spelled the same but with different meanings and pronunciations, corresponding to the two main pronunciations of "oo" discussed in Sample Homework #130. (The less common variant – where "roof" means the sound a dog makes -- occurs in the sentence.)

• *Addition With Carrying.* It is in the very last addition problem of the Sample Homework – "18 + 14" -- that we get to the meat of this landmark session. We have done a great deal of work on the base ten place system. Now we encounter our first important application of our new knowledge: addition with carrying![13]

o *Low-Key Approach.* Undoubtedly you are chomping at the bit to tie this new kind of problem, addition with carrying, into our work with the base ten place system! While your zeal is commendable, you must try to ratchet back a notch – we want to work our way gently from the concrete and mechanical to the more theoretical. In other words, we want to take small steps. So, proceed through the Sample Homework as usual until your child reaches the last addition problem, "18 + 14." This is the big new challenge – adding "8 + 4" in the ones column yields "12," which does not fit neatly in the ones column of the answer.

" . . . we want to work our way gently from the concrete and mechanical to the more theoretical."

Digression For Parents Only: Correct Carrying Procedure. Before we discuss how to tackle this new challenge with your child, it is probably

12 The answer is that to the left of the decimal point, zeroes are assumed for all columns to the left of the left-most non-zero digit, while to the right of the decimal point zeroes are assumed for all columns to the right of the right-most non-zero digit.

13 Some people insist on calling this procedure "regrouping" rather than "carrying." Pay no attention to them; you can be pretty sure that they are Communists.

worthwhile to summarize the proper "carrying" procedure. The correct procedure is to "carry" the "1" to the top of the tens column, like this:

$$\begin{array}{r} {}^{1} \\[-6pt] 18 \\ \underline{+14} \\ 2 \end{array}$$

Note that we use a very small "1" for the carried digit – this helps keep it separate from the original problem (which will be important later in certain multiplication problems) and also sends a gentle signal, not yet explicit or perceived, that someday we will eliminate writing the carried digit entirely and simply remember it. We often explain to the child that we write the carried digit smaller so that it is lighter and easier to "carry" all the way to the top of the tens column.

After we carry the "1" to its true home in the tens column, we add the numbers in the tens column and obtain "3," giving the correct answer of "32."

$$\begin{array}{r} {}^{1} \\[-6pt] 18 \\ \underline{+14} \\ 32 \end{array}$$

o <u>*Back to Business: Initial Matter-of-Fact Approach to "18 + 14."*</u> Eventually, we will establish some theoretical connections between carrying and our work on the base ten place system. But at this early stage, we do not want to get too theoretical!

Instead, we will take a matter-of-fact approach, quickly teaching your child the mechanical procedure for solving carrying problems. Once she is comfortable with this procedure and proud of her ability to do carrying problems, we will begin to introduce theoretical connections to our place system work.

First, allow your child to try this problem on her own, without warning her that it has a new twist. She will probably be puzzled by the two-digit answer for the ones column addition ("8+4"). There are many possible scenarios – the most common is that after a little thought, she will write "12" below the ones column and then add the tens column to get "2," ending up with "212" as follows:

$$\begin{array}{r} 18 \\ \underline{+14} \\ 212 \end{array}$$

If she does this, or takes another wrong approach, simply correct her in a matter-of-fact way, without making a big deal about it – perhaps along the following lines:

Sample Transcript: Initial Approach to Carrying

"Whoops, hold on a minute. Let's look at that again. [Rewrite the problem, 18+14, in a blank area on the Sample Homework sheet.] Go ahead and do it again, and I'll show you something. Start with the ones column, just like always – just like you did before. [Let your child begin, and intervene just as she is about to write '12' as the answer for the ones column.] OK, petunia,[14] here is the cool new trick. We are going to do something new, called 'carrying.' Have you ever heard of carrying? Well, I think you will like it! When you get '12' for the ones column, you need to write the '2' down here [point to the ones place in the answer area], but we have to CARRY the '1' up here, like this [write a small '1' in the correct place as follows:]

$$\begin{array}{r} {}^{1} \\ 18 \\ +14 \\ \hline 2 \end{array}$$

See how we use a little '1' so that things do not get crowded up there? Also, it is easier to CARRY a little '1' all the way to the top like that. A big '1' would be a lot heavier! We have to put the '1' up there because that is where it lives, in the tens column, and it has to get in line with all the other numbers in the ten column!" [Now erase the '1' that you wrote and have your child write a small '1' in the appropriate place.]

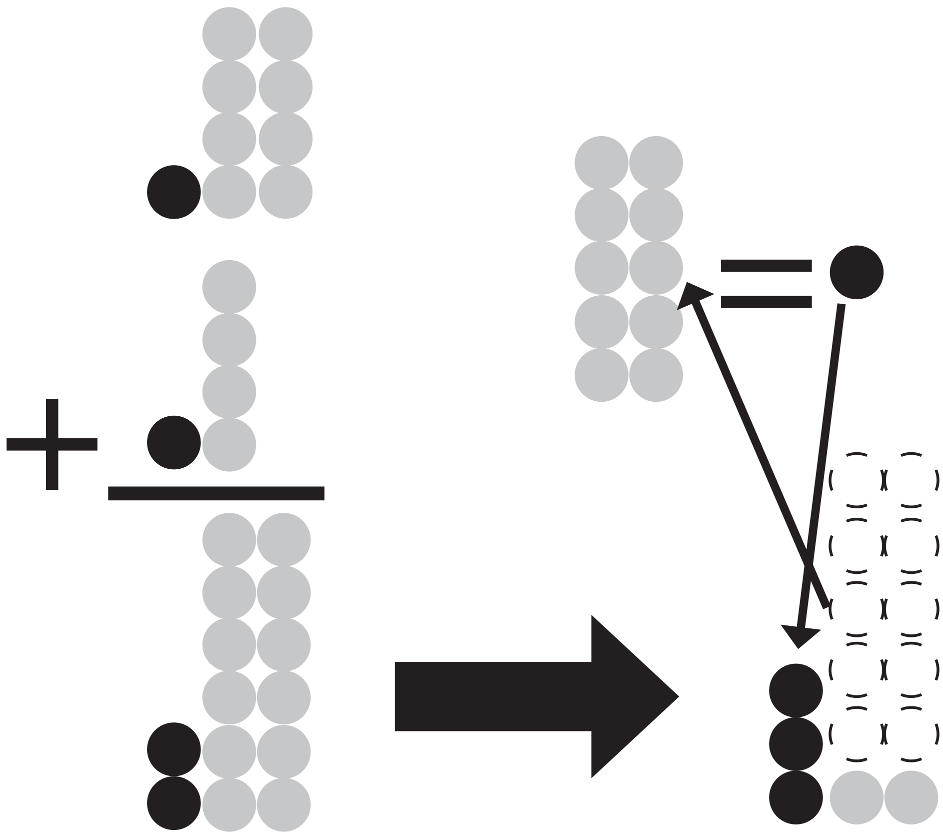
Now that the '1' is in the tens column where it belongs, you can just go ahead and add up the tens column. Make sure that you add the '1' we carried! [Point with your child at each of the three 1's in the tens column if necessary.] What do we get when we add these three ones? That is right – three! So, just like we always do, write the '3' where it belongs in the answer. And there you go – '32' is the correct answer! Well done! You have done a CARRYING problem! Yippee!"

o *Check the Answer Using Old Techniques*. You and your child want to make sure that this new "carrying" method works! So, ask your child to check her answer using the old "call yourself" method – i.e., she should call herself "18" and count up

14 You should of course feel free to substitute your own term on endearment in lieu of "petunia."

fourteen numbers using her fingers. She should get the correct answer of "32." Ask her to check the answer a different way -- perhaps using pennies (i.e., count out two piles of pennies (18 and 14), push them together and count the combined pile).

o *FieldWork*. There is a very nice way to capture the idea of "carrying" using coins (this technique was devised by an alpha test group parent). For example, to do "18 + 14," one simply begins with one dime (black circles below) and eight pennies (gray circles) and adds one dime and four pennies, converting ten of the pennies into a dime at the end:



15. ***Sample Homework #132.*** Do the usual place system work on at least some of the numbers in the second row of the Sample Homework. Remember to use the "stepping stone," one-step-at-a-time approach (see the discussion of Sample Homework #118, above) to the multi-syllable words in the third line and the sentence. And, of course, take your time with the two addition problems with carrying ("16+16" and "18+12"), building on the work done in the previous session. You might also want to try the following with, say, "18+12":

• *A Parental Error Illuminates a Few Interesting Ideas*. For fun, tell your child that you too are going to do the problem, '18+12.' Then do it as follows (i.e., incorrectly):

$$\begin{array}{r} 18 \\ \underline{+12} \\ 210 \end{array}$$

Follow this "error" up with a discussion something like the following:

Sample Transcript: A Parental Error Illuminates Carrying

"Whoa! I'm not sure that is right! That answer seems way too big, doesn't it? Let's check it – can you check it for me? [Have your child check it with the old "call yourself" method.] Yikes! You get '30' and I think that is the right answer. So how did I make such a big mistake?

Somehow I ran into trouble when I added 18 and 12 column by column, right? Let's go back over it and see what was different or weird. Which column did I add first? That's right, the ONES column! Very good. So let's do that again. Could you go ahead and help me add the ones column?" [Be patient while your child works on this.]

And what do you get? That's very good – ten. That is right! That is what I got too, ten. So where do we write the 10? Can we put the number 10 in the ones column of the answer?" That's what I did. [Discuss and listen, then write 10 in a blank area of the Sample Homework sheet or on another piece of paper.] Well, let's look at 10, let's think about 10 – are we ALLOWED to write '10' in the ones column? Or is part of it in the ones column and part in the tens column? Which column is the '0' in? Right, the ONES column! Very good. And which column is the '1' in? Right, the TENS column! Excellent! So we cannot write the number ten just in the ones column, can we?

[Go back to the addition problem, "18+12."] We added "8 + 2" and got "10." So, how do we write down the '10?' First, where does the '0' go? We'll worry about the '1' in a minute – where does the '0' go? Which column do we put the '0' in? Very good! Yes, the '0' goes in the ONES column. So go ahead and write the '0' in the ones column of the answer.

[It will, of course, look like this:

$$\begin{array}{r} 18 \\ \underline{+12} \\ 0 \end{array}$$

Make sure your child writes the '0' down directly under the '2' – firmly in the ones column.]

This is fine, isn't it, because we are finished with the ones column; we already added the ones column, so we are done with it and we can write down the ones column of the answer. So it is OK to write down the '0' where it belongs.

Now, in which column does the '1' go? In the number '10', what column is the '1' in? [Point to the number '10' which you wrote separately. [15]] YES, VERY GOOD – the '1' belongs in the tens column! So we will want to put the '1' in the tens column of the answer, won't we?

[Go ahead and write the '1' in the tens column of the answer, like this:

$$\begin{array}{r} 18 \\ \underline{+12} \\ 10 \end{array}$$

Then pause for a moment, and say . . . .]

WAIT A MINUTE! We cannot do that! We cannot write the '1' in the tens column of the answer, because we have not added that column yet! We have not done the tens column yet! So we cannot write down the tens column answer yet! What can we do? [Pause and discuss.]

Here is what we do. Is that '1' very heavy? Is it a really big, heavy number? No, of course not! So it will not be very hard to carry. Let's just carry the '1' up to the very top of the tens column, like this:

$$\begin{array}{r} {}^{1} \\ 18 \\ \underline{+12} \\ 0 \end{array}$$

15 Why do we write down a "10' separately from the problem to use as a reference? It turns out, for reasons unknown (at least to us), that old skills are often forgotten or not employed in new contexts. For example, in working on word problems, a child who can easily add 28+42 as a number problem may well have trouble solving that same problem when she extracts it from a word problem and writes it down. Here, we are separating the place system discussion of the number "10" from the new context of carrying so as to (i) help your child remember the proper place system treatment of "10" and (ii) help your child connect her old place system knowledge to this brand new area of carrying.

Do you see the '10?' We split it up, but here is the '1' and here is the '0' [point to each]. Notice that we write the '1' very small, to make it easier to carry – and also to help us remember that we carried the '1' up there – it was not there to start off with. [Now have your child finish the problem.]"

16. *Sample Homework #133*. For a nice change, you may want to have your child just read the numbers in the second row of this Sample Homework, and do your place system work with the three numbers following the phrase "Place System" in the fifth line.

17. *Sample Homework #135*. Note that your child has the opportunity to write the word "not" in the sentence (so that it reads "I am not Sammy."). Of course, as a general matter your child is always welcome to write on the Sample Homework sheet! Unlike even the best television shows, we seek active engagement, not passive watching. [16]

18. *Sample Homework #136*. Another great little story, motivated by Sammy's description of herself as the "Queen of Light and Air." Also, did you notice the landmark step taken in this Sample Homework? Yes indeed, congratulations to all you discerning parents who noticed that we have dropped the rows of letters. We are finished with the letters! (If you wish, you can celebrate this landmark with your child in a small ceremony.)

19. *Sample Homework #138.*

- *"Ing" Sound*. Make sure to discuss the "ing" sound and its connection to action words (verbs).
- *Carrying With Three-Digit Numbers.* Notice the extension of our carrying work to problems involving the addition of three-digit numbers. Approach this in a matter-of-fact way, but be prepared for some difficulties. Often, a child will perceive this as an entirely new task, rather than as a straightforward extension of the two-digit problems. That is OK! Take your time and approach it in the same fashion that we introduced two-digit carrying problems (see discussion of Sample Homework #131).
- *The "Free-Will" Fraction Exercise.* Ask your child to read the fractions and then use the shapes to illustrate the fractions. She may choose which shape goes with which fraction (this is where the "free will" comes in), but she should draw a line from the fraction to the shape. If she is firing on all four cylinders,[17] she will probably use the triangle for "½"!

20. *Sample Homework #140.* Eat-n-Park is a chain restaurant, much like a Denny's, that routinely wins the annual award for the best family restaurant in Pittsburgh. Shockingly, Eat-n-Park manages this feat despite being in direct competition with Bob Evans Restaurants.[18] When discussing the intersections, you may

16 Here is our contentious statement for the day: There is no such thing as a good television show, any more than there is good heroin or good tobacco. If you want your child to be smart, turn off the TV and do the Plan with her.

17 Believe it or not, this phrase used to be "firing on all eight cylinders"!

18 Why are we surprised that Eat-n-Park can beat Bob Evans? Here in Pittsburgh, Bob Evans Restaurants routinely draw in customers

wish to refer back to the discussion of Sample Homeworks ## 76 and 78 in Chapter 12. Note that the only areas that are the intersection of three shapes are (i) the innermost circle in the second diagram, and (ii) the top rectangle in the third diagram.

21. *Sample Homework #141*. We introduce the thousands column! Point out that a comma is not required, but it is fun and makes things a little more clear. We use the same type of trick in reading four (and five and six) digit numbers that we used in reading three-digit numbers (see discussion of Sample Homework #118): cover up the three numbers to the right of the comma; read the remaining number that you see (i.e., the number to the left of the comma) and say "thousand;" then read the rest of the number as usual. Thus, for "1,001," we cover up the "001" and read the remaining digit – "one" – then say "thousand," and then read the remaining number "001, which is of course just "one." The result is "one thousand one."

22. *Sample Homework #142*. Our experience reveals that children enjoy these types of off-beat, slightly-out-of-kilter stories. Also, they send a message to the child that these sessions are fun, not serious and scary.

23. *Sample Homework #143.* Well, we have been moving at a very rapid pace, introducing three-digit numbers, the base ten place system, the geometric meaning of fractions, addition with carrying, and last session four-digit numbers – surely it is time to pause and consolidate for a while, right?

" . . . surely it is time to pause and consolidate for a while, right? I hope you see that this is a trick question."

I hope you see that this is a trick question. What about Galileo, and all that motivational stuff we discussed at the start of this chapter? Of course it is not time to coast! Think about the stories that children enjoy – stories about Harry Potter, or hobbits, or series of unfortunate events. Children (and adults) enjoy stories where every page has a new development, or a major new character, or some sort of ghoulish disaster. We learned about four-digit numbers? Yes we did; that was yesterday's excitement. TODAY, we learn about <u>multiplication</u>!

- *First, We Read the Multiplication Problem*. We begin with horizontal multiplication problems because they can be read left-to-right, just like ordinary language. Ask your child if this is an addition or subtraction problem. Discuss, mentioning that in fact you and your child are about to do something new and exciting! Then have your child read all four of the multiplication problems, pointing out that "·" and "x" both mean the same thing and are read the same way, as "times." By now your child should know that "=" is read as "equals" or "is." For example, the first multiplication problem should be read as, "Two times three equals …."

- *Introduce the Meaning of Multiplication – Just a Fancy Form of Addition.* Now, you and your child are ready to tackle the first multiplication problem: "2 · 3." We will introduce the process of mul-

with huge signs saying, "Try our Fried Mush!" How can anyone compete with that?

tiplication at the concrete level by using coins, pictures, and finally fingers. So make sure that you have eight pennies (or nickels, or dimes, etc. – the key is that you have eight of the SAME coins) and a blank sheet of paper. Then introduce multiplication as just a fancy form of addition, perhaps proceeding along something like the following lines (of course, as usual with these "transcripts," the following is just a guideline; please tailor the session to your own style and your child's preferences):

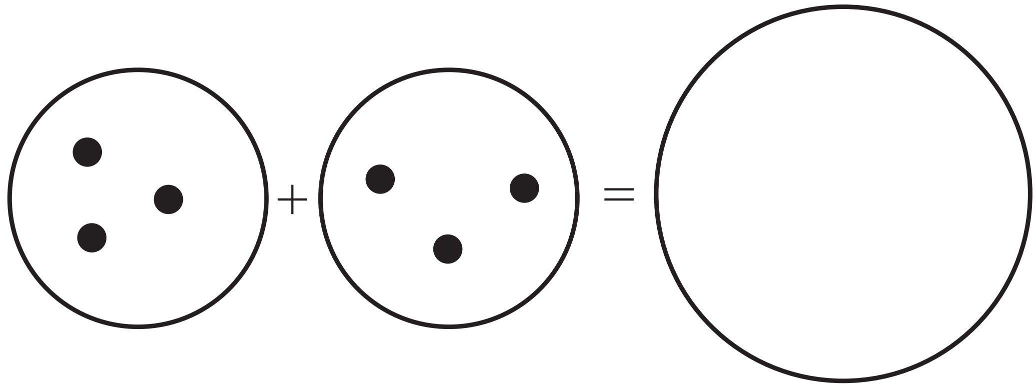
Sample Transcript: The First Multiplication Problem

"You know, multiplication is VERY advanced math – it is FOURTH GRADE MATH! It is probably way too hard for us to try, right? Because we have trouble with new kinds of math, right? [Hopefully, your child will disagree vehemently. If not, lead her into vehement disagreement. The point is that you and your child are math experts, math adventurers; you have been through countless math engagements and have emerged victorious in each case. There is no math challenge that you will not tackle!] OK, OK, we will go ahead and try it. But it is very hard!

So, 'two times three' – here is what that means – it means that we add two [point to the '2'] threes [point to the '3']. Does it mean that we add two plus three? NO, of COURSE not! That would be an addition problem, 'two plus three' – we would write that like this: '2 + 3=___.' (Go ahead and write out the addition problem next to the '2 · 3 = __' on the homework sheet. Point out the difference between the plus sign and the '·' multiplication sign.)

Instead, we are doing multiplication: 'two times three.' That means that we are adding two threes together. (Have your child cross out the '2 + 3 = __' that you just wrote.) Let's do that with pennies. (Have your child make two groups of three pennies, and then push them together and count them.) Yes, three plus three is six, of course, so two threes is six. So what is 'two times three'? Yes indeed, 'two times three' is two threes, so it is just six. Very good! We did our first multiplication!

Let's do this with a drawing, too. The multiplication problem is "two times three' – so we have to add two threes together. Let's draw two groups of three (do this, as below):



Here we are adding two threes. Can you count the total number of dots and fill in the answer with the right number of dots (i.e., six)? [This "dots-in-a-circle" approach was developed by one of the Plan's alpha test group parents.]

GREAT! Let's do this problem ONE MORE TIME! This time, we will use our fingers. We are doing 'two times three,' so we have to add two threes. [Hold up your left and right index fingers.] So we have one three here [left index finger] and one three here [right index finger]. And we add these two threes together [bring the two index fingers together]. We add three plus three. There, see how hard that is – you will NEVER be able to tell me what three plus three is! That's right! SIX! Very good. I guess maybe we can multiply! [19]

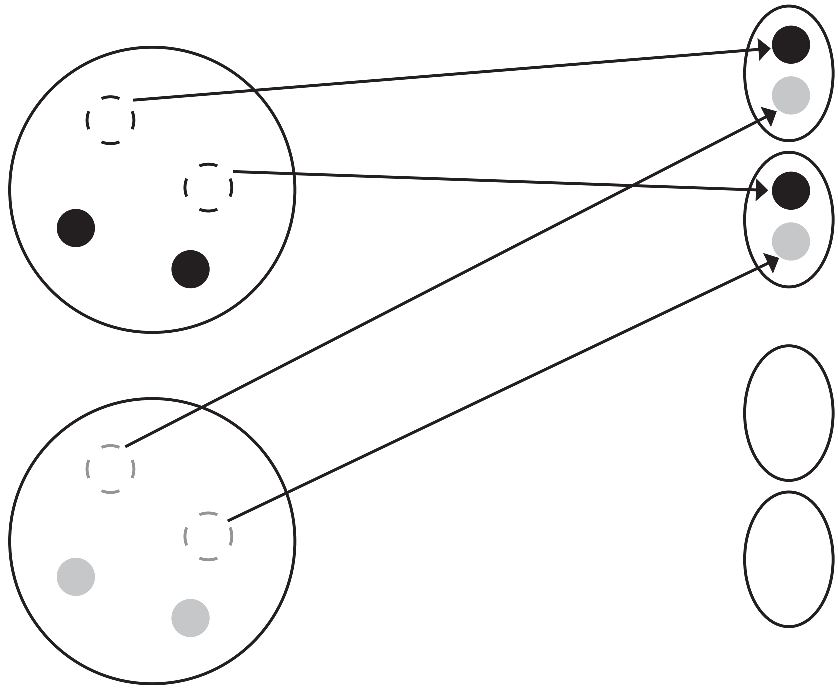
OK, let's try the next one – read it to me. Yes that is right, 'two times three.' Wait – haven't we seen this before? What does it mean? It means 'two threes' – we add two threes."

Continue on with the remaining problems. In each case your child should do three things: (i) read the problem aloud; (ii) tell you what it means; and (iii) come up with a solution. Feel free to use pennies, dots-in-circles, fingers, or even rote knowledge in arriving at solutions, as you see fit. You do not want to bore your child, so it would probably be overkill to use each method on each problem! But over time and across many problems, each method should be used at least occasionally.

• *Note to Parents: Commutativity of Multiplication*. Multiplication is "commutative" because order does not matter: "2x3" is exactly the same as "3x2." We will introduce this idea soon, but if your child notices it in the course of her work, use that opportunity to jump on it and discuss it.

- o Pennies are a great way of showing the equivalence of, e.g., "2 x 4" -- two groups of four -- and "4 x 2" -- four groups of two. Start with two groups of four pennies and construct four groups of two pennies by taking pairs of pennies, one from each group of four, one at a time until you have four groups of two pennies.

19 Why do we use one finger, instead of three fingers, to represent the number "3?" Because we are trying to help your child think of multiplication as the addition of <u>groups</u> of twos, threes, fours, etc. This is one reason why the "dots-in-a-circle" approach is so nice.



o Use the term "commutative" if this opportunity does arise, and draw a connection to the commutativity of addition ("2+4" is the same as "4+2"). Point out that subtraction is not commutative: "2-4" is very different than "4-2;" in fact, we don't even know what "2-4" means at this point! For now, though, unless you child notices this idea on her own, we will stick with a left-to-right reading of multiplication problems.

• *Multiplication Involving the Addition of More Than Two Numbers.* The final multiplication problem, "3x2," means "three twos" – i.e., "three groups of two" -- and thus involves the addition of three numbers. If necessary, write the addition down this way – "2+2+2" – and have your child work through it left to right, one step at a time (recalling our metaphor of crossing a river by using stepping stones).

24. *Sample Homework #144.*

• *"Ch" Sounds.* Note the emphasis upon the word-initial "ch" sound in the row of words.

• *Usual Procedure for Multiplication Problems*. Follow the usual three-step procedure for the multiplication problems: read the problem; say what it means; and then solve it. Again, it is best to use a variety of methods of solution: pennies; "dots and circles;" fingers; and recognition.

• *Commutativity of Multiplication.* Use the last multiplication problem, "4·2," to introduce the idea that the order of the numbers being multiplied does not matter, i.e., that multiplication is "commutative" (and you should use and teach this term). First ask your child to compare her answer for "4·2" with the answer she found for "2·4." Then write out a few more similar pairs of multiplication problems (e.g., "2x3" and "3x2," "2x5" and "5x2," etc.), and ask your child to do them and see if she notices anything. When she sees the pattern, ask her to articulate it as a rule. Point out the labor-saving aspects of knowing this rule (you only have to memorize half of the multiplication facts)! Consider the same idea in the case of addition (which is commutative) and subtraction (which is not commutative).[20] Explore with examples and pennies –see discussion of Sample Homework #143 above.

25. *Sample Homework #145.* Here we introduce the more common (and far more useful) vertical notation for multiplication problems. Take a minute or two to discuss the idea that the vertical and horizontal forms are just different names for the same thing – just as "Mary Samantha" and "Sammy" are different names for the same person! When your child reads the vertical problems aloud, she should read the horizontal line at the bottom of the problem as "equals."

26. *Sample Homework #146.* It may seem to you that we are moving very fast! But that is only because you are used to the standard American system of math education. We are actually moving at exactly the right speed.

• *Our Insincere Politically Correct Moment.* Nonetheless, if our fast pace has unsettled you at all, please know that we are sensitive to your concern, and regret any discomfort our rapid pace may have caused.

• *Borrowing!* Now, let us move on. In this homework, we introduce the ten-thousands column (which should be handled in the usual fashion, see discussion of Sample Homework #141) and – BORROWING!![21] This is a big deal! Make sure that your child understands that we are doing some very advanced work here!

The borrowing occurs in the two subtraction problems, "50-12" and "31-12." It is almost universal that the child will consider the ones column of "50-12" and write "2", thinking of "2-0" instead of "0-2." So, we have to be a little careful in our approach to these problems.

• *Initial Approach to Borrowing: Quick Concrete Solution Procedure (Three Easy Steps).* You might want to approach these initial borrowing problems in a quick, concrete, matter-of-fact way at first. For example, you could proceed as follows:

Sample Transcript: Initial Approach to Borrowing

20 As noted in an earlier footnote, subtraction can be viewed as the addition of a negative number; when viewed in this light, subtraction is commutative (but you must be careful what it means to reverse the order of a subtraction problem: "7 + (-2) = (-2) + 7;" not "7 − 2 = 2 – 7")!

21 Or, in the Communist lexicon, another form of "regrouping."

"Now, before we start the subtraction problems, let me tell you something exciting. These problems are a little different! They involve BORROWING! So, look at the first problem, "50-12," and see if you can see what is different. Let's start with the ones column, just like we always do. Can you read the ones column to me? [Your child should read 'zero minus two.' If she says 'two minus zero,' discuss it and let her correct it.]

"Do we EVER give up? [NO!!!!] "

Very good! 'Zero minus two.' Can we do this? Hold up zero fingers, and see if you can take two away from zero. [Discuss this for a while – concluding that 'zero minus two' does not really make any sense, because you cannot take two away from zero! [22]] So, what can we do? Do we have to just stop and give up? Do we EVER give up? [NO!!!!] No! Of COURSE not!

Here is what we can do. The ones column and the tens column are friends – they are part of the same number. So, why not have the ones column just 'BORROW' a ten from the tens column? Here is how we do that, in three easy steps:

o <u>Step 1: Borrow a Ten and Move It Into the Ones Column.</u> First, we borrow a ten from the tens column and move it into the ones column, like this:

$$\begin{array}{rr} \overset{4}{\cancel{5}} & 10 \\ \hline -\ 1 & 2 \end{array}$$

Because we borrowed one of the tens from the tens column, we had to change the "five" to a "four," since there are only four tens left in the tens column.

o <u>Step 2: Subtract the Ones Column</u>. Now, we subtract the ones column, "10-2":

$$\begin{array}{rr} \overset{4}{\cancel{5}} & 10 \\ -\ 1 & 2 \\ \hline & 8 \end{array}$$

22 What about negative numbers, you ask? It is WAY too early to introduce negative numbers – we will not do that for at least a few weeks! Seriously, if your child seems to be on the verge of coming up with some part of the idea of negative numbers in your discussion of "zero minus two," please pursue it – save the discussion of borrowing for another day, and simply deal with one-digit subtractions leading to negative numbers. The Plan treasures and encourages these moments of genius!

o <u>Step 3: Subtract the Tens Column</u>. Finally, we subtract the tens column, "4-1":

$$\begin{array}{r} \overset{4}{\cancel{5}}\,10 \\ \underline{-\,1\;\;2} \\ 3\;\;8 \end{array}$$

And we have our answer, "38."

• *Follow-Up, More In-Depth Approach to Borrowing*. Now, either in the first session involving this Sample Homework #146 or in a subsequent session, we should spend a little more time discussing the idea of borrowing. Remember, we are building CPUs, not storage banks! For example, we could tackle the second borrowing problem, "31-12," as follows:

Sample Transcript: More Borrowing

"Now let's do '31-12.' This problem also involves BORROWING! Let's start with the ones column, just like we always do. Can you read the ones column to me? [Your child should read 'one minus two.' If she says 'two minus one,' discuss it and let her correct it.]

Very good! 'One minus two.' Can we do this? Hold up one finger, and see if you can take two away from one. [Discuss this for a while – concluding that 'one minus two' does not really make any sense, because you cannot take two away from one!] So, what can we do? Do we have to just stop and give up? No! Of COURSE not! Instead, we BORROW, just like we did before. Remember our three steps? First, we borrow a ten from the tens column and put it in the ones column.

Since we are borrowing a ten from the tens column, we have to take one ten away from the tens column. So, we have to change the three tens to two tens, because we are taking one ten away when we borrow it. We do that like this [here you should be making the indicated changes in the problem on the Sample Homework sheet]:

$$\begin{array}{r} \overset{2}{\cancel{3}}1 \\ \underline{-\,12} \end{array}$$

But what do we do with the ten that we borrowed? Well, we put it in the ones column, because the ones column borrowed it from the tens column – these two columns are friends!. We squeeze the ten in like this:

$$\begin{array}{r} \overset{2}{\cancel{3}}\,11 \\ \underline{-\;1\;\;2} \end{array}$$

So, can you see what we did? [See if she can explain it.] Yes, that's right, we just borrowed a ten from tens column and gave it to the ones column. How many tens were there in the tens column when we started? [Three] Yes, that is right – three, because we started with a three in the tens column. But now that we borrowed a ten from the tens column, how many tens are left in the tens column? [Two] Yes! There are only two tens in the tens column, because we borrowed one. But, we did not lose the ten we borrowed! No way! Instead, we put it in the ones column. Now, instead of a 'one' in the ones column, we have an 'eleven' in the ones column.

Do we usually do this? No, usually we keep all of our tens in the tens column. We only do this when we need to do borrowing!

Now we can do the problem! First do the ones column – tell me aloud what you are doing. ['Eleven minus two is nine.'] Excellent – so we write '9' in the answer in the ones column.

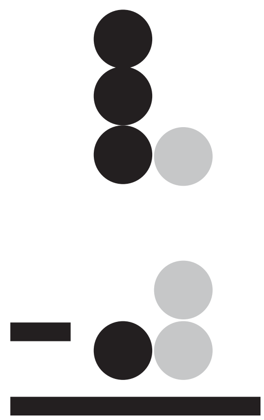
$$\begin{array}{r} \overset{2}{\cancel{3}}\,11 \\ \underline{-\;1\;\;2} \\ 9 \end{array}$$

And of course now we can do the tens column. Do we say 'Three minus one?' No, of course not -- we have to use the two, not the three! We scratched out the three! ['Two minus one is one.'] Good, so write the '1' in the answer in the tens column:

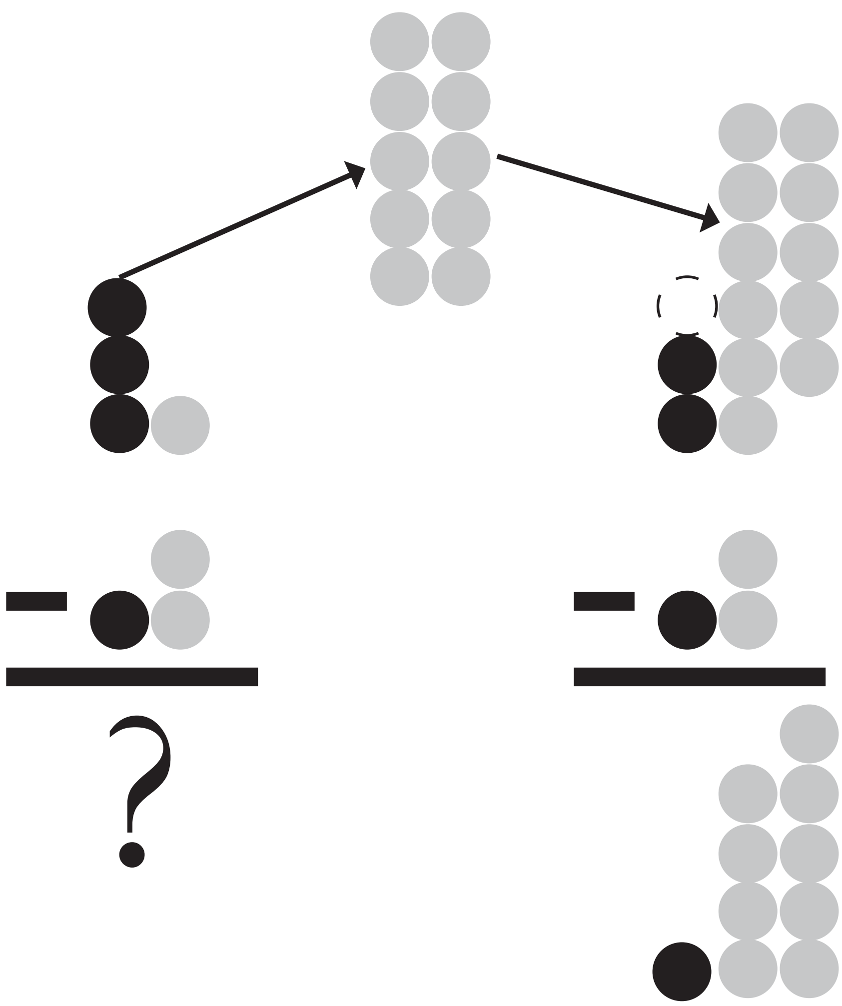
$$\begin{array}{r} \overset{2}{\cancel{3}}\,11 \\ \underline{-\;1\;\;2} \\ 1\;\;9 \end{array}$$

And our answer is – just read it off like we always do – 'nineteen!' Very well done. We did a BORROWING problem!"

• *Field Work.* There is a very nice way to capture the idea of borrowing using coins (again, this technique was devised by an alpha test group parent). For example, to do "31 – 12," one simply begins with three dimes (black circles) and one penny (gray circle) and tries to take away one dime and two pennies:



But, there are not enough pennies in "31" to allow removal of two pennies. So we "borrow," exchanging one dime for ten pennies and putting them where they belong, in the pennies column. Then the subtraction is easy:



• *Brainwashing.* Note that the story in this Sample Homework is a rather blatant effort to brainwash your child. Feel free to modify this if, for example, you want three grandchildren rather than only two.

27. *Sample Homework #147.* Again, use the three-step method described in the discussion of Sample Homework #146 to work the two borrowing problems. Note that "20-3" is a borrowing problem – if it helps, your child can think of this as "20-03," with a "hidden zero."

28. *Sample Homework #148.* This story may require a bit of explaining – for example, do you know what a "levee" is? You may wish to play the song ("American Pie" by Don McLean) for your child.

29. *Sample Homework #149.* Here we take a brief vacation from borrowing, and instead focus on fractions. Ask your child to write three fractions – her choice! – in the spaces provided to the right of "¼." Then, have her pick one, not too easy, and divide and color the circle to show that fraction. For example, if she picks 5/8, she should divide the circle into eighths and color five of the eighths.

30. *Sample Homework #150.* This would be a good time to reinforce and further explore the idea that multiplication is commutative (see discussion of Sample Homework #144), an idea that will be very helpful when we tackle multiplication problems involving a factor of five in the next few Sample Homeworks.

• *A Concrete Example of Order-Independence.* After your child works out "2x3" and "3x2," ask her to look at them and see if she sees anything interesting. Prompt her as necessary to focus her attention on the fact that the order does not matter; the answer is the same regardless of which factor comes first and which comes second. I.e., multiplication is commutative.

• *A Labor-Saving Rule!* Then move on to the vertical multiplications, "4x2" and "2x4," and once your child has solved the first one ("4x2"), point out that she does not need to work out the second one ("2x4"), because we know the answers are the same. She can simply write down the answer to the second problem, "8," without any further thought or work! This is a labor-saving rule!

• *Every Multiplication Can Be Read In Two Ways* Another important way of stating the idea of commutativity is that every multiplication can be read two ways. For example, "2x4" can be read to mean "two fours" or, alternatively, "four twos." Similarly, "5x7" could be thought of as "seven fives" or as "five sevens." Discuss this with your child, and practice reading multiplication problems both ways.

" . . . every multiplication can be read two ways."

• *Often, One Way of Reading a Multiplication Problem Is Easier To Solve Than the Other Way.* There is another way in which the commutativity of multiplication is a labor-saving device: it allows us to choose the easiest version of a particular problem. Perhaps you have already noticed that often one way of thinking about a multiplication problem is much easier to solve than the other way. For example, if I think of "5x7" as "five sevens," I have to add sevens, which is not particularly easy (especially for a young child): 7, 14, 21, 28, 35. But, if I think of "5x7" as "seven fives" -- which is perfectly permissible, since order does not matter -- I can simply count by fives (seven fives) to get the answer – 5, 10, 15, 20, 25, 30, 35. Because of the Field Work, your child should find counting by fives fairly easy!

Or, consider "68x2" – most people find it much easier to think of this as "two sixty-eights" ("68+68") rather than as "sixty-eight twos":

2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+
2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2+
2+2+2+2+2+2+2+2

Perhaps you agree! In any event, you should discuss this idea with your child as you approach each multiplication problem, asking, "OK, which is the easier way to think of this problem?" (Usually, it will be easier to work a problem of the form "AxB" or "BxA," where A is greater than B, by thinking of it as "B A's" – e.g., it is easier to add up "six eights" than it is to add up "eight sixes.")

• *Summary.* To recapitulate, the commutativity, or order-independence, of multiplication is a labor-saving device in two ways:

> **" . . . the commutativity, or order-independence, of multiplication is a labor-saving device . . . "**

o <u>*Workload Reduction:*</u> Your child can get away with memorizing only half of the multiplication facts (e.g., "2x6"), and use the rule of commutativity to know the other half (e.g., "6x2"); and

o <u>*Ability to Choose The Easier Reading of a Problem*</u>*:* Your child can choose which way she prefers to think about the problem, which is satisfying at multiple levels (ease of solution, empowerment, etc.).

31. *Sample Homework #151.*

• The story is actually the beginning lyrics from a song by Ed McCurdy, perhaps most famously sung by Simon and Garfunkel.

• As we begin to do multiplications involving a factor of five, your child's Field Work in counting by "5's" will come in handy. Hearkening back to your work on commutativity in the preceding homework sessions, discuss with your child the idea that it does not matter whether the "5" is the first or second factor. Either way, we can do the problem by counting by "5's." For example, we can solve "2x5" and "5x2" each by counting two fives: "5;" "10."

32. *Sample Homework #154.* Make sure to focus your child's attention on the peculiar letter group "igh."

33. *Sample Homework #155*. This is a landmark Sample Homework, one that extends the Pittsburgh Plan's program in an entirely new direction.

For more than three years, and through hundreds of sessions, we have been pursuing what one might call a "leading edge" goal: the very rapid development of advanced reading and math skills. This ambitious program has been based upon two fundamental ideas: (i) we can trust our children – our children are able learners who can achieve great things if we simply give them the opportunity and then get out of their way; and (ii) large distances can be covered over time if we just stick with a consistent program of three to four short session a week. And we have made great progress using this approach.

However, at this point, we realized that Sammy had a significant problem. Although she had become very accomplished in the areas of reading and math, she had reached the age of five without being able to say her "r" sounds at all. "Girl" was "guhhl," "red" was "wed," and "car" was "cahw." Her friends had begun to make fun of her, and her special enrichment sessions at kindergarten were no longer focused on math and reading, but instead – quite properly – were remedial sessions targeting her word pronunciation.

We consulted an expert, who told us that the problem might still resolve itself, but that it would soon be time to start a speech therapy program. This worried us an several levels. First, we know a number of very able adults who exhibit the same "r"-based speech problem, suggesting that there was a very real possibility that neither time nor professional intervention would cure the problem. Second, we worried that professional speech therapy sessions might have two unintended consequences: (i) Sammy might internalize a view of herself as a child with a significant problem – the opposite of self-esteem; and (ii) Sammy might be labeled as a "remedial" case in her academic and social environments. It is a sad fact that, once acquired, labels are hard to remove in our educational system. For example, in many school systems, once a child has been labeled as gifted and placed in the school's gifted learning program (perhaps in second grade), that child will not be removed from the gifted program for the rest of her academic career, regardless of performance! [23]

> **"It is a sad fact that, once acquired, labels are hard to remove in our educational system. "**

23 Of course, one would not wish upon any child the emotional consequences of being eliminated from a gifted program, but that just exhibits the absurdity of the whole idea of such a program – why would we wish upon any child the emotional consequences of being labeled as "not gifted" from the outset, either? Why do we need this arbitrary system of labeling at all? It is becoming increasingly clear that (i) there are many forms of intelligence, (ii) an individual's intelligence scores can vary widely depending upon the particular type of intelligence being measured and the type of measurement instrument (written test, oral test, time-sensitive or not, etc.) being used, and (iii) in general, the population varies continuously, in a bell-shaped curve, across each intelligence scale. It is strange enough, in light of these complexities, for school systems to draw a thick black line dividing children into two arbitrary groups labeled as "gifted" and "non-gifted;" it is even stranger that a seven-year-old child who is recognized as "gifted" will continue to be so recognized all the way to age eighteen without any further evaluation along the way!

Once again, we thought long and hard, and once again, we came up with a solution – the same solution! We decided to tackle Sammy's "r" problem by (i) trusting the child[24] and (ii) working consistently and relentlessly over time. Sound familiar? Of course it does; it is just the Pittsburgh Plan, but this time applied to a trailing edge problem rather than a leading edge adventure.

• *A Big Caveat.* There is no reason at all that the Pittsburgh Plan's methods should work in a remedial context, particularly one involving speech production! Remember, the Plan's approach is premised on the idea that children possess an intrinsic inductive learning ability – often characterized as a language-learning ability -- that can achieve great things if we simply give it the opportunity. Fair enough, but what happens if that language-learning ability is not functioning properly? Can we still rely on it? In Sammy's case, we were facing a developmental problem, a departure from the normal effective functioning of her language-learning ability. And, quite possibly, the "r" problem was unconnected to the cognitive functions that the Plan generally addresses. In short, there were at least two theoretical problems in applying the Plan's methodologies to Sammy's "r" problem: (i) we ran the risk of trying to use a broken tool to fix itself (i.e., relying on an impaired language-learning ability to fix a language problem); and (ii) we were applying a methodology relating to cognitive functions to a problem that might well be physical or least might involve very different mental functions.

In fact, Sammy cured her "r" problem, but we cannot establish a causal link between our efforts and the resolution of the problem. Perhaps all that was needed was time and a bit of physical maturing. We are taking a few pages to describe our use of the homework sessions for remedial purposes simply because it seemed as if our approach worked, and if you are as concerned about a problem as we were about Sammy's "r" problem, perhaps you will want to try a similar approach. After all, if you keep things low-key and non-pressured, there is probably not much of a downside!

• *And Hence a Clarification.* Remember at the very outset of this book when we said that even if the Plan is all wet, it cannot possibly hurt to add an hour a week of relaxed, non-pressured parent-child time to the schedule? In tackling Sammy's "r" problem, we were not relying at all on the Plan's inductive learning approach. In fact, in a sense we were not really relying on the Plan at all. Instead, we were hearkening back to something that the Plan uses but that is far more elemental – to the very basic notion that <u>if you want to get something done, you do</u>

> **" . . . if you want to get something done, you do two things: (i) you put it in the hands of someone who really cares about it; and (ii) you tell him to stick with it through thick and thin, to never give up. "**

24 Given our energetic criticisms in earlier chapters of experts in child educational theory, it is probably not surprising that we chose to trust the child rather than the experts in tackling this problem!

<u>two things: (i) you put it in the hands of someone who really cares about it; and (ii) you tell him to stick with it through thick and thin, to never give up.</u> That is how the Sistine Chapel was painted; that is how the light bulb was invented; that is how a person runs a four-minute mile. And that, my friends, is how a child beats an "r" problem.

• *A Summary of Our Approach to Sammy's "R" Problem*. Here is how we approached Sammy's "r" problem:

o *<u>Calm, Frank Discussion of the Problem.</u>* First, because we trust the child, we discussed the problem with Sammy. We made sure she understood the nature of the problem (a tape recorder helped), but also treated it in a very calm, matter-of-fact fashion. This was just another task, like learning addition or taking out the trash.

o *<u>Calm, Frank Discussion of Our Approach to the Problem.</u>* Next, we discussed with Sammy our plan of attack. We made it clear that, as with all of our endeavors, we would take our time with the "r" work, and that we certainly did not expect quick results. We would work on "r" sounds in the homework sessions and also in our Field Work, and we would enlist sisters and brothers to assist us.

o *<u>Analysis of the Problem</u>*. We then read available sources on pronunciation of "r" and examined as best we could the way in which we produce the "r" sound. We concluded as follows, and discussed these conclusions with Sammy: [25]

- There are several different "r" sounds;
- The most difficult 'r" sound for Sammy appeared to be the guttural "r" sound produced deep in the throat, often following "g," as in "tiger," "girl" and even the growling sound a bear makes: "grrr;"
- Word-initial "r" sounds ("rabbit," "run") produced in the front of the mouth were much easier for Sammy to produce and in fact seem to be significantly different sounds than a guttural "r" – possibly with significantly different production mechanisms;
- Word-ending and middle-of-the-word "r" sounds that are not guttural ("far," "larger," etc.) seemed to occupy an intermediate position, both in terms of area of production (middle of the

25 We do not pretend to be experts in speech therapy! We only present our approach because (i) paper is cheap, and you can easily skip it if you are not interested, and (ii) it actually seemed to work, when we have seen many cases where expert treatment did not. We do not want to oversell what we did and what we know – which are, respectively, not clear and very little – but, on the other hand, our distrust of psychological and educational expertise has grown to such giant proportions that we actually suspect "not clear" and "very little" could serve as the gold standards of these fields!

mouth) and difficulty for Sammy;

- Production of "r" sounds, especially guttural "r" sounds, seems to involve a lifting up and side-to-side curling of the back of the tongue;

- One way to encourage this type of tongue action is to ask the child to use the front tip of her tongue to flick air backwards down into her throat, almost scraping along the roof of her mouth, as she attempts to make an "r" sound; and

- Mostly, "r" production would require a lot of tongue experimentation by Sammy as she tried to say "r" words, focusing on the back of her tongue.

o *Approach to the "R" Task in the Homework Sessions*. We agreed with Sammy that we would be very consistent in doing "r" tasks in the homework sessions and in our Field Work, and that she would do a lot of experimenting with the back and middle of her tongue as she worked on "r" sounds. Then, like the wind that scours the cliffs, like the slow drip of water that forms a stalactite, like . . . OK, OK, I'll get on with it! We began our persistent but gentle approach by asking Sammy to read and practice the "r" words at the bottom of this Sample Homework (in case you have forgotten, we are discussing Sample Homework #155).

o *A Peek At the Last Page*. In the end, after many homework sessions, a great deal of Field Work, and about nine months of persistent effort, Sammy walked into her father's office one morning with her older sister, who said, "Dad, listen to Sammy say 'girl'!" Sammy then said, after a dramatic pause, "Guhhl." After a brief minute, Sammy and her sister burst out laughing and Sammy said, "girrrl." It was by all reports a very nice moment.

" . . . we are working on exclamation points in this Sample Homework!!! How exciting!!!"

34. *Sample Homework #156*. Notice that the introductory words, "More 'R' practice," are themselves "r" words to be practiced!

35. *Sample Homework #159.* Sammy had been asking a lot of repetitive questions immediately before we created this Sample Homework – payback is sweet!

36. *Sample Homework #161.* Note that we are working on exclamation points in this Sample Homework!!! How exciting!!! We also stick our toes in the water of several new math ideas.

• *Multiplying By One*. Use the two multiplying by "1" problems (3x1 and 10x1) to explore the special result you get when you multiply a number by "1" – i.e., nothing changes; you get the same number back that you started with! If you start with some given number, say "11," and multiply it by any number other than "1," you change it, but multiplying by "1" does not change anything; you still have "11." Work out a few extra examples of multiplication by "1" to reinforce this special rule.

o *The "Identity" for Multiplication*. Because multiplying a number by "1" does not change the number, we call "1" the "identity" for multiplication, or the "multiplicative identity."

o *Commutativity.* Because multiplication is commutative, it does not matter whether "1" comes first or second (whether we multiply by "1" from the left or from the right); either way, it still acts as the multiplicative identity. Work a few examples to reinforce this.

o *Exploration: the Additive Identity*. For fun, explore the idea of "identity" – i.e., a number that does not change anything when it is used in a mathematical operation like multiplication, division, etc. – a little further by considering addition. Is there an additive identity? Is there some special number that does not change anything if we add it to another number? For example, is there some number that we can add to "3" and still have "3?" (Of course, "0" fits the bill and is in fact the additive identity. See the discussion of Sample Homework #50, above.)

• *Multiplying By Zero*. Now, consider the final multiplication problem: 0x4. Ask your child to work it (talk her through it as necessary – e.g., "suppose I have four zeroes, 0+0+0+0 – what do I have?"). Then discuss it, perhaps as follows (remember that we do not ask rhetorical questions – always wait for an answer):

"We know that zero is the additive identity. Is it also the multiplicative identity? Can we multiply numbers by "0" and not change the numbers? [No, of course not, "1" is the multiplicative identity.] What does happen if we multiply a number by zero? We always get zero! Zero is sort of the exact opposite of the multiplicative identity – when we multiply a number by zero, we do not get that same number back; instead, we always get zero back!" [Finish by asking your child to state the rule for multiplying by zero in her own words.]

37. *Sample Homework #162*.

• *Reading Larger Numbers.* Here, we introduce some larger numbers. Remember to use the procedure described earlier (in the Discussion of Sample Homework #141) in reading these numbers.

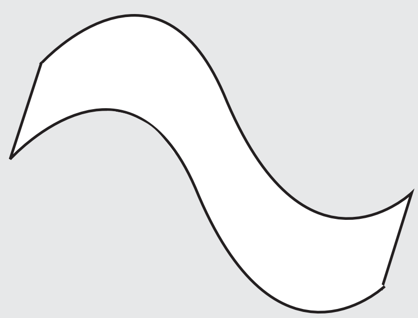
• *Three-Dimensional Shapes.* Also, we introduce our first three-dimensional shapes: a cube, a cylinder, and a rectangular prism (like a cube, only having some faces that are not squares). These new shapes, along with our old friend the parabola, are labeled in the Sample Homework.

• *Dimensions.* You may want to use this Sample Homework as an opportunity to discuss the idea of dimensions. It would be unrealistic to expect your child to grasp fully the abstract concept of a one-, two-, or three-dimensional object at this early age – that will probably have to wait at least a few months! In fact, most adults do not have a fully fleshed-out idea of the meaning of dimensions.

Brief Sidebar: Discussion of Dimensions for the Parent. Perhaps we should spend just a minute or two discussing the idea of dimensions. In truth, this is a very difficult area! At the most basic level:

• *One-Dimensional Shapes.* A one-dimensional shape is like an idealized string, having length but not width or height. Examples include a line, a parabola, and the boundary (but not interior) of a circle or a square.

• *Two-Dimensional Shapes*. A two-dimensional shape (e.g., a square or circle, including the interior) has length and width but not height. In other words, a two-dimensional shape has "area." If one ignores the (tiny) thickness of a sheet of paper, the sheet is a two-dimensional shape, and most or all two-dimensional shapes can be constructed out of a sheet of paper by cutting, folding and/or bending the sheet. At an elementary level, one often says that two-dimensional shapes such as a square or a triangle can be "drawn on" a sheet of paper or a wall, but that ignores the case of two dimensional shapes that are not flat but instead are embedded in three-dimensional space in a curved fashion, such as, for example, the cylindrical surface of a drinking glass or a sheet of paper with a bend in the middle:



• *Distinguish Between a Shape and a Picture of a Shape.* Note, by the way, the difference between drawing an actual shape – such as a triangle – on a piece of paper and drawing a picture of a shape, such as a cube or a cylinder, an a sheet of paper. In the case of a triangle, which is two-dimensional, we can actually say that there is a triangle on the sheet of paper. In the case of a cube, which is three-dimensional, we cannot say that there is an actual cube on the sheet of paper – instead, there is only a picture of a cube on the paper.

• *Three-Dimensional Shapes.* A three dimensional shape such as a cube or a sphere (a ball) has length, width and height; i.e., it has "volume." Everything in the real world is three-dimensional; even a sheet of paper has a tiny height! One- and two-dimensional shapes are abstract concepts; three-dimensional shapes can be touched, felt, and picked up.

o *A Better Definition of Dimension For Parents.* We know – We know! – that we should not do this, but we cannot help ourselves. We have to take just a minute to give a better definition of dimensionality. Please feel free to ignore this paragraph! The best way to think about the number of dimensions of an object is to say that the number of dimensions equals the number of separate pieces of information (or "coordinates") that is needed to locate a point on or inside the object.

• For example, if you want to identify a point on a piece of string, you only need to specify one coordinate; you need only supply one piece of information: the distance along the string from the left end to the point, for example. Thus, a string is one-dimensional.

• If you want to specify a point inside a circle or square, you need to specify two coordinates (e.g., "x" and "y" in a standard Cartesian coordinate system). Likewise, if you want to specify a point on the surface of the Earth, you must specify latitude and longitude – again, two coordinates. Thus, a circle, a square and the surface of the Earth are each two-dimensional shapes.

• If you want to specify a point inside the Earth (such as the location of buried treasure), you must specify not only latitude and longitude, but also how deep below the surface of the Earth one must dig to get to the treasure. Likewise, latitude and longitude will be sufficient to specify a particular building, such as for example West Holmes Hall on the campus of Michigan State University, but you also need to know the correct floor to find the pizza party! In other words, you need a third coordinate —height. So, the Earth itself (not just the surface of the Earth, but the entire Earth) is a three-dimensional object.

Why is this "number of coordinates" approach the best way to think about dimensionality? Because this approach is easy to generalize and extend to more than three dimensions, which can be very important in mathematics and physics. It is not clear what one could add to length, width and height to describe a fourth or fifth spatial dimension, but it is certainly easy enough to just specify additional coordinates. Instead of (x,y,z), one simply writes (x,y,z,w)!

• *Introductory Discussion of Dimensions With the Child*. I guess that was more than just a "brief sidebar" for parents! Hopefully it provided you with some insight into the depth and importance of the concept of dimensionality. But now it is time to return to our task at hand, which is to introduce the idea of dimensions to your child. Again, we have no expectation that she can master the kind of analysis described above at this time! But we do hope to toss a few ideas out into the general environment and let her begin to get familiar with them. A good way to begin is simply to ask your child how the new three-dimensional shapes, the cube, cylinder, and rectangular prism, are different from the two-dimensional shapes in the top row of shapes, i.e., the square, circle, triangle and rectangle. Discuss this for a while – you can look at the discussion of special topic homework 162A below for ideas to throw out into the discussion if you wish. Then, ask your child how the (one-dimensional) parabola is different from all the other shapes. Feel free to use the terms "one-dimensional," "two-dimensional," and "three-dimensional" in this discussion. At the most basic level, it is useful just to get your child thinking about these questions. If you would like to provide a bit more guidance, you should feel free to borrow a few discussion ideas from the following paragraphs, which provide the basis for a more detailed approach to dimensions.

• ***Special Topic Homework #162A.*** If you wish to dive into the topic of dimensions in more detail, either now or later, you may wish to use the next Sample Homework page, which is labeled "Special Topic #162A." (This could be done in the same session as Sample Homework #162 or in a different session – your choice!) You may wish to approach Special Topic #162A along the following lines:

Sample Transcript: Introduction to Dimensions

o *Introduction. "Now we are going to tackle a SPECIAL TOPIC! This special topic has to do with SHAPES. We are going to talk about three kinds of shapes – really, three FAMILIES of shapes. Here are the three families of shapes: [hold up one finger] one-dimensional shapes; [hold up two fingers] two-dimensional shapes; and [hold up three fingers] – can you guess the name of the third family of shapes? That's right, THREE-dimensional shapes.*

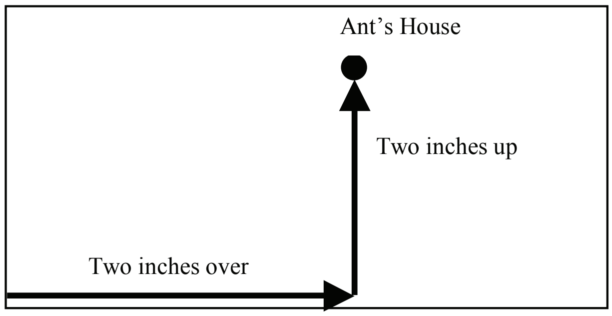
Here on the worksheet [again, you are using the Special Worksheet labeled Special Topic #162A], you can see some examples of the first family of shapes – one-dimensional shapes. Can you trace your finger along a few of the one-dimensional shapes? Good! Can you tell me about these shapes? What are they like? How are they like each other?"

o *Ideas to Launch Into the Discussion*. Continue on in this vein, soliciting your child's thoughts and providing a few ideas and suggestions. You may wish to draw from the following ideas for this discussion:

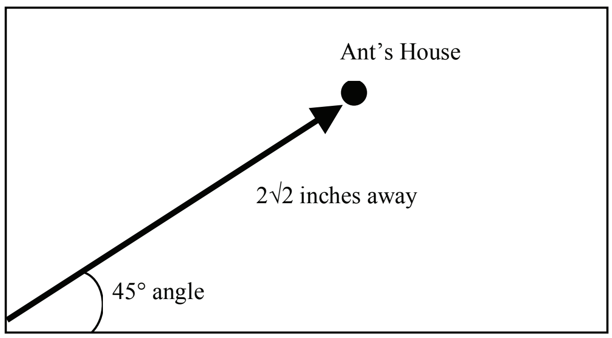
- *One-Dimensional Objects.* One-dimensional objects are basically straight or curved lines. A string or thread is pretty darn close to a one-dimensional shape, although of course it does have width and height if you look closely enough. True one-dimensional objects have length but no width or breadth. You cannot really color a one-dimensional shape because there is no "inside" – no interior – to color. (You may want to discuss the idea that the parabola does not really have an interior unless you close off the top of it and make it a two-dimensional shape.) If a tiny little ant lives on a string (put a little dot on a line to show the ant's house), we only need one number to give its address – all we need to know to find the ant's house is how far it is from the end of the string. (Trace along the string to find the ant's house.)

- *Two-Dimensional Objects.* Two-dimensional objects are shapes that have length and width but no height. They can be drawn on a piece of paper, or cut out and pasted on a wall. (We are ignoring the complications of embedding two-dimensional shapes in a three-dimensional space, as discussed briefly above.) You can color the inside of a two-dimensional shape. If an ant lives inside a two-dimensional shape such as a rectangle (put a dot in the interior of a rectangle to show the ant's house), we need to know two things to find the house: for example, measuring from the bottom left corner, how far over; and how far up.

(Demonstrate.)



Or, if we want to start at a corner and draw a line straight to the ant's house, we still need to know two things: which direction to draw our line (the angle); and how far to go along that direction. (Demonstrate.)



• *Three-Dimensional Objects.* Three-dimensional objects are shapes that have length, width and height. They cannot be drawn on a piece of paper, or cut out and pasted on a wall. If you want put a three-dimensional shape in a book, you have to use a pop-up book! We can put a picture of a three-dimensional shape in a book, but we cannot put the actual shape inside a book – it will pop right out of the page! You can put water inside many three dimensional shapes. If an ant lives inside a three-dimensional shape such as a cube (i.e., a box) (hold your finger at a location inside an actual box to show the ant's house), we need to know <u>three</u> things to find the house: how far over, going left to right; how far back, going front to back; and how far up, going bottom to top. (Demonstrate.)

38. ***Sample Homework #163.*** Notice that we have introduced a six-digit number! The same reading technique (see discussion of Sample Homework #141)will still work with this number. Ask your child to state the names and dimensionality of the shapes (note that we have added a sphere to our group of 3-D shapes).

39. Sample Homework #164. The special "r" work continues, of course. Meanwhile, the story consists of the questions most commonly asked by Sammy at this age. And, note that we have added a new row of shapes: triangular pyramid (3-D), diamond, and vertically oriented rectangle. See if your child can spot the relationship between the diamond and a square (the diamond is just a rotated square).

40. Sample Homework #166. It is much easier to do "two tens" than "ten twos," and to do "two nines" rather than "nine twos"!

41. Sample Homework #167. Here we recommend a brief session with flash cards and then introduce – as an advance outpost -- the multiplication of a two-digit number by a one-digit number. For now, we will keep it simple, but soon we will introduce carrying into these more complicated multiplication problems. You may wish to approach this new type of multiplication as follows:

- *Read and Analyze the Problem.* Ask your child to read the problem (e.g., "two times twenty-three"), and then ask her what are the two ways of thinking about this problem ("two twenty-threes" or "twenty-three twos"). Which way is easier? ("Two twenty-threes.")

- *Do the Problem Using Addition*. Then, ask your child to do the problem the easy way, as "two twenty-threes." Help her as necessary to write the problem down as an addition problem:

$$\begin{array}{r} 23 \\ \underline{+23} \end{array}$$

Your child is familiar with this type of addition problem, and may well be able to solve it easily, getting "46." However, be prepared for the possibility that she will have trouble doing this addition. It is quite normal for a child to have trouble extending a well-understood skill, such as the addition of two two-digit numbers, into a new area – in this case, as a way of solving a new type of multiplication problem.[26]

If your child experiences this type of difficulty, do not make a big deal about it! Stay calm and help her connect this new context to her preexisting skill at working these types of additions. For example, if she looks confused, you might say:

26 We might go so far as to say that we welcome this kind of confusion! After all, our primary goal is not to teach particular math skills; our goal is to help your child become a better thinker. And, as we have discussed at great length, this means that we want to help your child incorporate new knowledge into a mental framework that contains a vast number of connections. Here we have a golden opportunity to forge a new set of connections.

Sample Transcript: Multiplication as Addition

> *"Now, we have done a great job of handling this new type of multiplication problem, multiplying a TWO-DIGIT NUMBER! You know what we have done that is so powerful? [As always, pause for a response, and listen to your child if she has something to say.] We have taken this new type of problem, and we have turned it into an old kind of problem! We turned it into our old friend, an addition problem! So now we can relax, because we know how to add two numbers like this, don't we? [Pause, etc.] Do you remember how to add these types of numbers? Do we add them by starting with the ones column and adding it first? [point.]"*

With this kind of discussion, your child will soon be able to complete the addition problem. Feel free to discuss how this is one example -- a powerful example! -- of our stepping stone approach to hard math problems (see discussion of Sample Homework #118, above).

• *Introduce Powerful New Technique.* Now comes the fun part! After your child has struggled through this slow procedure to get the answer -- "46" – she is primed and ready to learn a powerful new labor-saving technique. You might proceed as follows:

Sample Transcript: The "Trick" of 2-Digit Multiplication

> *"Now that was pretty easy, and you did a GREAT job – but I am going to tell you a secret. [Lean forward and whisper conspiratorially.] Are you ready for a secret? We can use place systems to do this problem in a VERY EASY way. You will be AMAZED! Here is a VERY FUN way to do '2 x 23.'*
>
> *[In a blank area on the homework sheet, or even on a separate sheet of paper, write the problem again in the usual vertical style:]*
>
> $$\begin{array}{r} 23 \\ \underline{\text{x}2} \end{array}$$
>
> *Here is what we do – we just use the place system. Can you tell me what number is in the ones column for '23?' That's right – 'three.' And what number is in the tens column of '23?' Yes – 'two.'*
>
> *So here is what we do. We start with the ones column and we do it first. We just ignore the tens column for a minute, and we just do*

the ones column. Help me read the ones column part of the problem while I point. [point as you and your child read.] Three . . . . times . . . . two . . . equals …. [27] Very good, that is right! The ones column problem is just 'three times two.' But what is three times two? [Give your child some help if she freezes or cannot remember how to do this, as will often happen when we introduce new areas – for example, remind her that 'three times two' means three twos or two threes; she should pick whichever is easier for her.] Yes! '3 · 2' is two threes, so it is six. So, since that is our answer for the ones column, we write that answer – '6' -- in the ones column of the answer. Will you do that please? [Help if needed.]

$$\begin{array}{r} 23 \\ \underline{x2} \\ 6 \end{array}$$

Very good – see how it works? [Point as you talk, pointing to the 'x' for 'times' and to the horizontal line for 'equals.'] Three . . . . times …. two … equals …. six. Excellent. That was certainly easy, wasn't it?

But now we have to do the tens column! This will probably be MUCH harder, right? Let's read it together. [Point as you and your child read.] Two . . . . times . . . . two . . . equals …. What does two times two equal? That's right – FOUR! Good! Since 'four' is our answer for the tens column, let's go ahead and write it down in the tens column of the answer. Can you do that please? [Help if needed.]

$$\begin{array}{r} 23 \\ \underline{x2} \\ 46 \end{array}$$

Very good! And look – we got the right answer! We got '46,' just like before. But this was a very easy way to do the problem, wasn't it?"

Continue on with the second new multiplication problem, "34 x 2." Just do this one the easy way.

42. ***Sample Homework #168.*** A very famous guest makes his first appearance on our show! Yes, for the first time, we meet the number "one million." Point out the need for two commas, and teach a reading trick similar to that which we used for thousands (see discussion of Sample Homework

27 Or, "two times three" -- since multiplication is commutative, either way is fine.

#141) – cover up everything to the right of the left-most comma, read the number that you see, say "million", then proceed as before to read the rest of the number.

- *Example*. For example, we would read the number 24,287,931 as follows:

  o Cover up everything to the right of the left-most comma (leaving "24,xxx,xxx"), and read the number you see – "twenty-four;"

  o Say "million;"

  o Cover up everything to the right of the next comma (leaving "24,287,xxx"), and read the number you see (ignoring the "24," because you have already handled it) – "two hundred eighty-seven;"

  o Say "thousand;"

  o Read the rest of the number – "nine hundred thirty-one."

- *Sound-Alike Words*. Also, notice our focus on words that sound alike but have different meanings. Discuss this concept with your child as she reads "right," "write," and "rite."

43. *Sample Homework #170.* Whew! We have been working hard, and in the process we have covered a lot of territory. Soon we will pause and consolidate our gains. But first, in this Sample Homework, we establish one more outpost – we introduce multiplication with carrying! We will introduce this topic and then abandon multiplication entirely for three Sample Homeworks! Starting with Sample Homework #177, we will reintroduce multiplication, then multiplication of a two-digit number (Sample Homework #179), and finally we will return to multiplication with carrying in Sample Homework #181. Most parents resist – or even resent! – this non-linear approach, but it is designed to mimic the way in which language is learned. As we have discussed in many contexts, it is our hope that by adopting this type of tactic we can enlist the full power of your child's inductive learning mechanism – often referred to as her innate "language learning ability" – to help her learn math as well as she does her native language.

- *Introduce Multiplication With Carrying Concretely and In a Matter-of-Fact Way.* Although we often discuss theoretical ideas, or even ask interesting questions that we may not answer, it is important to remember that in introducing new ideas we want to begin by giving your child a solid concrete (some might say "operational") foundation. Once she can apply the correct procedure to solve a problem, her comfort level with the new idea will allow her to think about it and begin to build connections and bridges from her old knowledge to the new idea.

- *Example*. Thus we approach the first new problem (25 x 2) in a casual, relaxed fashion, perhaps proceeding along the following lines:

Sample Transcript: Multiplication With Carrying

"OK, let's do the first multiplication problem, 25 x 2. This is going to be a lot of fun, because it involves something new – as you will see! Go ahead and start – and tell me what you are thinking as you work. [Your child will say 'five times two is . . . ten.] Very good, how do you write down the ten? [Once in a great while, a child will do it correctly, carrying the '1' as described below. If so, praise her and skip the rest of this paragraph. Usually, however, she will try to squeeze the whole '10' into the ones column of the answer.] Whoops, we do not want to put the whole '10' down there! We can put the '0' down there, but we have to carry the '1', just like in addition. [Point to where the '1' should go, see below.]

$$\begin{array}{r} {}^{1}25 \\ \underline{\times\ 2} \\ 0 \end{array}$$

[Discuss this if your child seems to desire a discussion, but otherwise move on – this is just an outpost.]

Now, the fun part is the next step. We don't add the '1' that we carried yet. First, do the tens column multiplication – go ahead, and tell what you are doing. [Help your child as necessary to say 'two times two is …. four.'] Very good – but can we write down the '4' yet, or do we have to add the '1' that we carried to the tens column? That's right, we have to add the '1.' So, if we have '4' and we add the '1', what do we get? ['5'] Right!"

$$\begin{array}{r} {}^{1}25 \\ \underline{\times\ 2} \\ 50 \end{array}$$

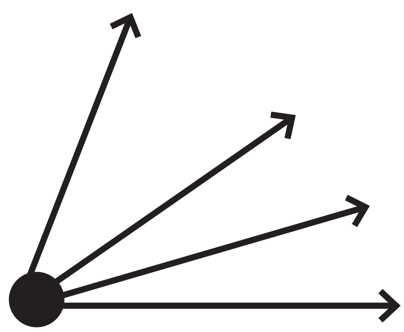
[Point out that this answer makes sense if we think about money, because two quarters makes fifty cents. Then proceed with the remaining problems – note that the next one, 31 x 3, does not involve carrying; it is a breather before the new materials resumes.]

44. ***Sample Homework #174.*** Since we are consolidating on most other fronts, we introduce three new geometrical ideas: line; ray; and line segment. Note that the line goes forever in both directions, the ray goes forever in one direction (like a ray of sunlight), and the line segment does not go forever in any direction – a line segment has a length, unlike a ray or a line.

45. ***Sample Homework #176.*** Now we introduce the idea of an angle. It has been our experience that this concept should not be defined in the abstract (that is very hard to do – try it!), but should instead be defined operationally (concretely). In other words, we will teach your child how to draw an angle, and give her examples of angles (big angles, small angles, etc.), and let her construct her own inductive definition of angle. In fairness, I should mention that our presentation of angles in Sample Homeworks ##176 and 177 is merely an outpost, and we will not return to the full treatment of angles in this book, which is nearly at an end. The full treatment of angles occurs in the first few weeks of the next course, described in the planned follow-up volume to this one, Newton Ascendant: The Pittsburgh Plan, Part 2. Or, of course, you should feel free to create your own presentation of angles at the time of your choosing!

- *Building An Angle*. Point out that you draw an angle – sometimes we like to say "build" an angle – by drawing two rays from a common starting point. Ask your child to draw (build) a few angles of her own, helping as necessary.

- *Enrichment*. If your child is interested, you or she can draw more angles and explore the idea of the size of an angle -- of what is a big angle or a little angle. You could start with a horizontal ray and draw bigger and bigger angles from it, one after another, as follows:



You might even think about what is a zero angle! (Two rays right on top of each other.)

46. ***Sample Homework #177.*** Note that we have two interesting multiplication problems, "6 ·1" and "8 · 0." Use these two problems to discuss two general rules of multiplication, multiplication by '1' and multiplication by '0.' (You may wish to refer back to the discussion of these rules in connection with Sample Homework #161 above.) For the two columns at the end of this Sample Homework, ask your child to draw a line connecting the geometrical object to its name. (For fun, see if she puts arrows on the end of the "line" she draws.)

47. ***Sample Homework #178.*** The second row of words illustrates one of the many complexities of the English language -- the multiple pronunciations of the letters "ough."

48. ***Sample Homework #179.*** Here and in Sample Homework #180, we introduce an outpost for another major idea: adding fractions! Here is one approach to the first addition of fractions problem, "½+ ½":

- *Read the Problem.* As always, begin by reading the problem: "One half plus one half equals. . . ."

- *Common Sense Approach (Demystification).* Then, read the problem a few more times together, emphasizing the words "one": "ONE half plus ONE half." Ask your child if she knows the answer. If not, say, "OK, well then, what is ONE tiger plus ONE tiger? [TWO tigers.] ONE cookie plus ONE cookie? [TWO cookies.] ONE ball plus ONE ball? [Two balls.] ONE half plus ONE half? [TWO halves.]"

- *Draw A Picture.* Ask your child to draw a circle and divide it into two halves. Then pointing as you talk, ask her "How many halves do we have if I have ONE half [point to one half of the circle] plus ONE more half [point to the other half]?" [Count them if necessary – you have TWO halves.]

- *Key Requirement: The Same Denominator.* We can only add two fractions if they have the same denominator. We cannot add one tiger plus one lion, we cannot add one apple plus one orange, and we cannot add one half plus one third – at least not yet! Soon (well, soon in the usual Pittsburgh Plan progression following the end of this book . . .) we will learn some tricks to handle different denominators. But for now, we can only add fractions if they have the same denominator.

    - *The Basic Three-Step Procedure.* If we have two fractions with the same denominator, such as 2/3 + 1/3, we add them in three simple steps:

    o First, the answer will be a fraction, so get ready by drawing a fraction line where the answer will be (as shown by the arrow).

$$\frac{2}{3} + \frac{1}{3} = \frac{}{}$$

    o Second, write the common denominator as the denominator of the answer.

$$\frac{2}{3} + \frac{1}{3} = \frac{}{3}$$

o Finally, add the two numerators and write the sum as the numerator of the answer.

$$\frac{2}{3} + \frac{1}{3} = \frac{3}{3}$$

For the remaining two addition-of-fractions problems, have your child verify that the two fractions have the same denominators and then apply the above three-step basic procedure to get a solution. Of course, at this point we are not focusing on simplifying fractions (e.g., we are not concerned in the above problem – yet! -- that 3/3 = 1).

49. *Sample Homework #182.* The only new wrinkle in this Sample Homework is presented by the multiplication problem "43 x 3," and then again by the following problem, "61 x 2." In each of these cases the child must figure out how to handle a two-digit result of the multiplication in the tens column. For example, in the problem "43 x 3," after multiplying "4x3" to get "12," your child may try to carry the "1" to the hundreds column as follows:

$$\begin{array}{r} {}^{1}\,43 \\ \underline{\text{x}\ \ 3} \\ 29 \end{array}$$

This is fine! Simply point out to her that because there is nothing in that column, the "1" just falls right on through to the hundreds column of the answer:

$$\begin{array}{r} {}^{1}\!\downarrow 43 \\ \underline{\text{x}\ \ 3} \\ 129 \end{array}$$

Alternatively, you can just point out that since the "4" is the last number, there is no need to carry anything – simply write the entire answer to "4x3" (12) in the answer column immediately.

50. *Sample Homework #183.* DIVISION!! Yes, that is right, in this Sample Homework we introduce division, a topic that generally is delayed in the schools until third or even fourth grade! But why delay? After all, division is just multiplication backwards!

- *"78x2."* First, however, before introducing division, we warm up by doing a difficult multiplication problem ("78x2"), which should be handled carefully and slowly. Just to review, the problem should be done in the following steps:

o *Ones Column.* First, the ones column multiplication ("8x2") yields "16." We write down the "6" in the ones column (because we are finished with the ones column) and carry the "1" to the tens column (because we have not done the tens column yet).

$$\begin{array}{r} \overset{1}{7}8 \\ \underline{\times\ 2} \\ 6 \end{array}$$

o *Do the Tens Column Multiplication and Add The Carried "1".* Next, we do the tens column work: "7x2" equals "14," but we must also remember to add the carried one after doing the multiplication (14 + 1), giving a grand total of "15." We write down the "5" and, then realizing that there is no work to do in the hundreds column, write down the "1" as well.

$$\begin{array}{r} {}^{1}\ \overset{1}{7}\,8 \\ \underline{\times\downarrow\ \ \ 2} \\ 1\,5\,6 \end{array}$$

• *Division.* Having completed a difficult warm-up problem, we are ready to tackle division. You may wish to approach this new idea as follows:

o *Pep Talk.* Give the usual pep talk about how advanced and fun division is – how most children do not get the chance to do division until they are in third or even fourth grade. Create an air of adventure and excitement. Perhaps remind your child of the standard idea that we tackle new ideas in stages, and do not expect immediate understanding (see Detailed Suggestion #2, in Part 3 of Chapter 10, above – as we state there, "our whole idea is to learn ideas over time rather than the first time they show up").

" . . . we tackle new ideas in stages, and do not expect immediate understanding . . ."

o *Read the Problem.* As always, we take our time and learn how to read the new type of problem. In this case, "4 ÷ 2 =" is read as "four divided by two equals," where the new symbol "÷" is read as "divided by."

o *Interpret the Problem.* What does "four divided by two" mean? For now, it means "How many twos are in four?" So, for a while, every time we read a division problem we will go on to restate it as a "How many …" question. For example, "6 ÷2" means "How many twos are in six?" [28]

o <u>*Solve the Problem*</u>. How many twos are in four? There are at least two ways we can approach this -- both of which should be discussed with your child. And, both approaches should be used frequently in this and subsequent Sample Homeworks.

- *Counting Approach*. First, we can figure out how many twos are in four by counting by twos, ticking off each count with a finger: "two, four …" -- and we see we used up two fingers, so there are two twos in four. If we are doing "9 ÷ 3," we can count by threes: "Three; six; nine" – giving us an answer of three.

- *Inverse Multiplication Approach*. Or, we can use our knowledge of multiplication, because division is just backwards multiplication (in a sense). For example, if we are confronted with "8 ÷ 2," we need to know how many twos are in eight. So, we think of a multiplication problem where some number times "2" equals "8":

  ___ x 2 = 8

  And, we try to remember what number would work – what number could fill in the blank to make the multiplication work. And that number – "4" – is the answer to our problem of how many twos are in eight.

51. ***Sample Homework #184.*** We return to the addition of fractions after a brief hiatus. Also, we introduce a new notation for division

$$2\overline{)6}$$

which, of course, is read as "six divided by two." We tackle division problems in this new notation exactly as we tackled the original form (using "÷"), interpreting the problem as meaning "How many twos are in six?" and proceeding from there.

52. ***Sample Homework #185.*** In the very bottom row, we introduce a third, very important notation for division – the fractional form of division. Thus, 4/2 is one way of writing "four divided by two." Again, we read these "fractional" forms of division problems carefully, to gain comfort with the

28 In the planned sequel to this book we give a second alternative interpretation of division, as a fraction. Thus, "4 ÷ 2" also means "What is one-half of four?" Apart from the many cognitive benefits of this type of alternative view of a mathematical operation, this approach has some very practical uses. For example, if the problem is "200 ÷ 2," it is easier to figure out "What is one-half of two hundred?" than it is to determine "How many twos are in two hundred?"

new notation, but our approach to solving the problem is exactly the same as before (see discussion of Sample Homework #183).

- *Three Notations For Division*. Note that we now have three different notations that we can use for division problems. For fun, every now and then ask your child to write a given division problem in the other two notations. For example, if the problem is written as "8 ÷ 4," she should write the problem in the following two additional ways:

$$4\overline{)8} \quad \text{and} \quad \frac{8}{4}$$

Note to Parents. This fractional notation for division is a big idea. Because now we see something startling – we see that fractions and divisions are just two different ways of thinking about the very same thing! We will explore this idea in great depth over the next few months – although not in the materials covered by this book, which is nearing an end, but instead in the sequel. For now, we confine ourselves to reading "improper fractions" – where the numerator is equal to or greater than the denominator – as division problems. Because this is such an important set of ideas however, which is not explored in the brief remaining pages of this book, let me provide a quick summary of our approach to any "fractional form" – i.e., anything that looks like a fraction, having a numerator and a denominator:

o *Fraction or Division Problem?* If the numerator is equal to or larger than the denominator, we think of the "fraction form" as a division problem. If the numerator is smaller than the denominator, we think of it as a fraction.

o *Division: Two Approaches*. If we decide we are dealing with a division problem under the above approach, we think of that problem in two alternative ways (e.g., for 26/2):

- *How Many …?* How many twos are in twenty-six?
- *What Is…?* What is one-half of twenty-six?

o *Fraction: Simplify.* If we decide we are dealing with a fraction under the above approach, we try to "simplify" the fraction using techniques that will be explained in the sequel (i.e., cancellation of common factors to reduce the fraction to its simplest terms).

53. *Sample Homework #187.* Note that in the problem "15 x 4," your child will have to carry a "2" to the tens column – the first time any number other than one has been borrowed or carried.

54. ***Sample Homework #189***. The very last problem of this Sample Homework, "24x12," introduces an important new kind of problem: the multiplication of two two-digit numbers.

- *Overall Approach*. We will, as always, begin by tackling this problem in a very concrete, heuristic way – we will simply do what it takes to solve it, procedurally, without a great deal of theorizing. Later, we can encourage your child to think about the underlying theory in discussions. But for now, we just want to add this type of problem-solving capacity to your child's tool set. Remember our approach: concrete foundations build confidence; confidence lays the groundwork for the construction of a theoretical edifice.

" . . . concrete foundations build confidence; confidence lays the groundwork for the construction of a theoretical edifice."

- *Four-Step Procedure*. Here is how we solve "24 x 12." First, just between we adults, we will think of it as being the sum of two problems, "24 x 2" and "24 x 10" (using the fact that "12" can thought of as "2 + 10"). This idea (which should not be communicated to your child at this early stage)[29] leads to the standard four-step procedure, which <u>should</u> be communicated to your child:

    o <u>*Step 1: The Ones Column.*</u> First, we do the ones column (24 x 2) in the usual column-by-column way, simply ignoring the fact that the bottom factor (12) is a two-digit number:

$$\begin{array}{r} 24 \\ \underline{\text{x}12\ } \\ 48 \end{array}$$

    o <u>*Step 2: Insert a Zero to "Keep Your Place."*</u> Now we are ready to tackle the tens column part of the problem (24 x 10). We begin by inserting a zero as show, to keep our place (since we are dealing with the tens column):

$$\begin{array}{r} 24 \\ \underline{\text{x}12\ } \\ 48 \\ 0 \end{array}$$

    o <u>*Step 3: The Tens Column.*</u> Now, without hesitation or fear, we proceed to do the tens column multiplication (24 x 1) in the usual column-by-column way, first "4 x 1 = 4" and second "2 x 1 = 2," writing down the answer to the left of the zero that we inserted:

29 The mathematically interested parent will perhaps realize from this that our standard procedure for solving the multiplication of two two-digit numbers is just an application of the distributive law: a • (b + c) = a • b + a• c.

$$\begin{array}{r} 24 \\ \underline{\times 12} \\ 48 \\ 240 \end{array}$$

o *Step 4:Add.* To finish our job, we add the two rows of answers and read the result as our answer:

$$\begin{array}{r} 24 \\ \underline{\times 12} \\ 48 \\ \underline{240} \\ 288 \end{array}$$

55. **Sample Homework #191**. This is the very last Sample Homework! To celebrate, let us do something special with the final problem: 1/2 + 3/2, Normally we would follow the basic three-step procedure outlined in the discussion of Sample Homework #179 in order to get the answer:

$$\frac{1}{2} + \frac{3}{2} = \frac{4}{2}$$

And, we would be content to leave it at that.

However, in this last Sample Homework, let us take one more step, integrating our work on adding fractions with our work on division in fraction form. Let us take the following final new step:

- *Simplifying an Answer that is an Improper Fraction*. Once we finish adding the fractions, if the numerator is larger than the denominator, we switch gears and think of the answer as a division problem. Thus, once we find that the answer is 4/2, we adopt a new perspective, and ask, "If we think of 4/2 as a division, can we get our answer in a simpler and better form?" And of course we can, because four divided by two is just two; our answer is just "2."

$$\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

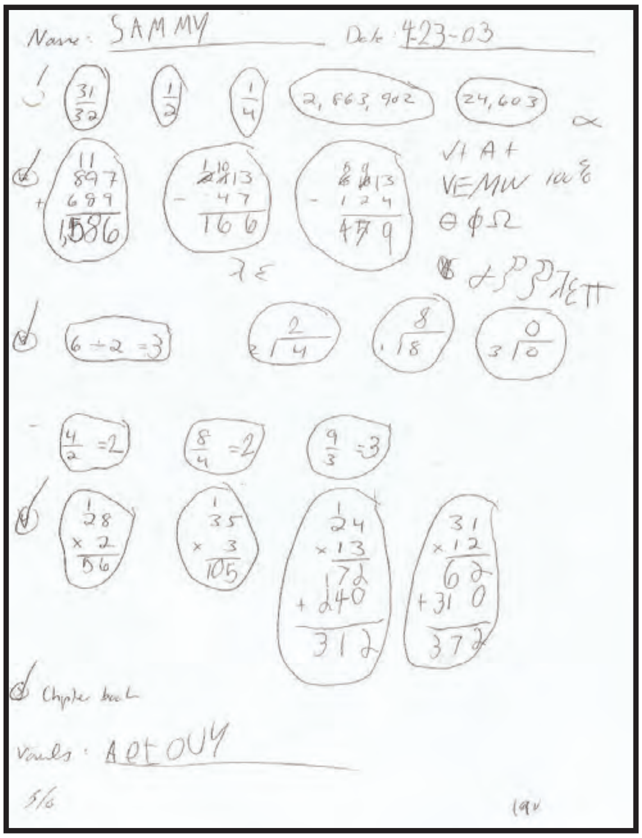


FIGURE 13.1: Fractions, seven-digit numbers, addition of three-digit numbers with carrying, subtraction of three-digit numbers with borrowing, three forms of division, multiplication of two-digit numbers with carrying, and reading from a chapter book – all in fifteen minutes, all by the same child who just turned six, all in a day's work for a participant in the Pittsburgh Plan!

PART 2: Field Work For Phase 5 – Independent Reading.

At this point, in Phase 5 of the Plan, there is only one type of Field Work that is critical, and that is independent reading. The importance of independent reading (which certainly can include reading to a parent or sibling) simply cannot be overstated. Do not be a prig about the nature of your child's reading material; for example, comic books are just fine! Within obvious large boundaries, anything that gets your child reading on her own for enjoyment is good. Also, give your child plenty of paper and many opportunities to write and draw – in particular, encourage her to write stories.

However, there are also a number of math-related types of Field Work that can add to the excitement and sense of importance surrounding the math topics that your child is learning so rapidly and well. For example:

- Add silver coins to practice adding mixed groups of fives, tens and twenty-fives

• Practice addition and subtraction problems using the "count up" method; [30]

• Practice addition and subtraction problems using mental math tricks (for example, add "29" to a number by adding "30" and then subtracting "1", or add a string of numbers by finding pairs that add up to "10," etc.);

• Practice counting by 2's, 3's, 4's, 5's and 10's;

• Use coins to do the types of field work described in the discussions of Sample Homeworks ## 131 (carrying), 142 and 143 (commutativity), and 146 (borrowing);

• Do fraction problems with a number of coins, blocks, etc. For example, take a group of twelve pennies and ask your child to divide it in half, etc.; and

• Discuss whether various things are two-dimensional ("2-D") – e.g., wall surfaces, sheets of paper, etc. -- or three-dimensional ("3-D") – e.g., balls, boxes, toys, etc..

30 This is the "mental math" method used by cashiers to make change. For example, "1000 – 263" can be calculated by counting up from "263" to "1000" as follows: "37" gets you to "300" and another "700" gets you to "1,000," so the difference is "37+700" which equals "737." Or, one can add "34 + 28" by first adding "20" to "34", getting "54," and then adding "8," yielding a final answer of "62."

PART 5: A Few Final Thoughts

What Next? Where do you and your child go from here? Read on…

Chapter 14: *More Worlds to Conquer*

"And when Alexander reached the sea, he wept, for there were no worlds left to conquer."

--**Ancient.**[1]

"Now this is not the end. It is not even the beginning of the end. But it is, perhaps, the end of the beginning."

--**Winston Churchill**, 1942.

Congratulations! You and your child have finished the Pittsburgh Plan for young children. It is fitting that we take a moment to contemplate the past few years and share a few valedictory remarks.

It is easy – and impressive! – to list the many math and reading skills that your child has developed over the course of the Plan. But that would miss the point; it would understate her (and your) achievements. The more lasting legacies of the past few years are your child's great ability to think and learn, her burgeoning confidence that she can tackle difficult problems and conquer them, and, perhaps most importantly, her joy in the thrill of such conquests! She has come to love learning; she has tested her strength and found that she is indeed strong.

But, is this the end? Having achieved so much, should your child now tread the more normal path of a standard elementary school education, with success assured because of her vast head start and her finely tuned learning engine? Or does this path seem somehow hollow to you?

Every valediction contains at least a few wistful strains of eulogy; every great triumph marks the end of a momentous struggle. And after completing a great struggle, we often find, like Alexander, that there was more meaning in the struggle than in the triumph.

"Every valediction contains at least a few wistful strains of eulogy; every great triumph marks the end of a momentous struggle."

So, if this is the end of your child's full-bore dash into the world of knowledge and cognition, then, like Alexander, we must not only praise your child's triumph, but also mourn the end of her time of brilliant achievement. But it is our hope that no such regret is necessary, that, like Churchill,

1 And probably apocryphal. A similar phrase is contained in McGuffey's New Fourth Eclectic Reader, Lesson XXXVI, William McGuffey (1866). It seems likely that this famous phrase originated in a misreporting of an anecdote from "On Contentment of the Mind," in Plutarch's Moralia, wherein Alexander wept when told by a teacher about the infinite number of worlds in the universe, saying: "There are so many worlds, and I have not yet conquered even one."

we may regard the completion of the Plan as merely the end of the beginning – the beginning of a life of curiosity and inquiry, of great effort and great reward.

Unfortunately, there is now a looming barrier to the further progress of your young Galileo, namely, the fact that she must spend more than six hours a day in an elementary school environment that has a very different approach than the Plan. How can you help your child continue to savor the joy of large challenge and great intellectual achievement in this new milieu? More generally, where do you go from here? What do you do next?

As discussed more fully below, we suggest that your child attend regular elementary school and that you enlist your child's teachers as your partners in helping her (i) take advantage of special programs (e.g., gifted learning programs, advanced reading programs, etc.), (ii) participate in reading and other enrichment activities, and (iii) achieve appropriate math placement, including advancement (in math only) where appropriate. We also recommend that you continue your parent-child homework sessions under the Pittsburgh Plan. This can be done, for example, by using the Pittsburgh Plan approach and techniques described in this book with problems drawn from either an above-grade math textbook from your child's school or one of the excellent books by Edward Zaccaro (e.g., Becoming a Problem Solving Genius or Challenge Math). Alternatively, there are several more levels of the Pittsburgh Plan itself – in fact, we have vast amounts of material covering all levels of math up to and including Pre-Calculus – and we are contemplating organizing those further materials for publication in a follow-up volume to this one, Newton Ascendant: The Pittsburgh Plan, Part 2. (Our decision as to whether to go forward with that second book will depend in part upon the extent to which this book finds an audience and in part upon the average energy level of our crack team of experts.) Of course, these are only suggestions; as always, you are the best judge of the appropriate course of action for your own child!

The remainder of this Chapter discusses a number of specific topics that are often raised by parents contemplating next steps.

1. Home Schooling Versus Regular School. Almost every parent who participates in the Plan for any length of time is tempted by the thought of home schooling his child. After all (the common argument goes), if so much can be achieved under the Plan in an hour or two per week, just imagine what could be done over the course of an entire school day!

The question of whether to home school a child is a personal one involving a wide variety of issues. For example, many parents who home school their children do so for religious reasons that are not necessarily relevant for other families. Also, the advisability of home schooling may vary depending upon the backgrounds of the parents – for example, two experienced teachers might well do a better job of home schooling their children than would two parents with backgrounds in other areas.

Leaving aside these types of special factors, from the perspective of a child's cognitive and social development, we do not generally favor home schooling. Why? Here is a partial list of reasons:

- *Elementary School Teachers Are Very Good*. Apart from the specific area of math, which can be handled separately as discussed below, we have found elementary school teachers to be very good – competent, admirable, and enthusiastic about their students. And this observation has held true over many years and numerous geographical regions. In other words, it would be difficult for the average parent to match the background, training and overall competence of a good elementary school teacher in the arena of educating young children.

"Often parents feel that a teacher is not treating or perceiving their child correctly, but in such cases it is generally the parents who are failing to see things clearly. "

Often parents feel that a teacher is not treating or perceiving their child correctly, but in such cases it is generally the parents who are failing to see things clearly. Parents are notoriously biased in favor of their children, while teachers usually have no particular ax to grind.

- *A Child Needs Honest Feedback, and Parents Cannot Be Relied Upon to Provide It*. In the usual scenario, the outside world provides a child with accurate feedback with respect to the child's actions, allowing the child to adapt and change to become more effective. Such feedback can come from teachers, fellow students – in truth, from any objective source. On occasion, this feedback can be tough to swallow, and the child can lean on the solid foundation of her parents and family, who love her unconditionally. This really is a pretty good system – accurate, objective feedback with a built-in safety valve! If one removes the objective teachers and fellow students from the equation, the critically important "feedback" function is jeopardized.

- *An Over-Involved Parent Can Stifle a Child*. You have been to children's sporting events; you know that over-involved parents generally have no clue that they are overdoing things. Can you imagine what it would do to a child to have such a parent in charge of her entire day, as a home school teacher? Yikes!

- *It Is Difficult to Duplicate the Important Social Aspects of Elementary School*. Eventually, our children must go out and live in the wide world. The necessary social skills are developed primarily in peer-to-peer interactions with a wide variety of other children – conditions that are hard to duplicate adequately outside of a school environment.

- *Home Schooling Is Very Hard For the Parents*. Parents matter too, and home-schooling has a gigantic effect on a parent's life.

Of course, the above discussion merely sets forth one view of home schooling. It should be recognized that there is certainly room for argument here -- that there are also clear advantages to home schooling. For example, it appears that home schooled children on average outperform their regular school peers on standardized college admissions tests such as the SAT. The mere fact that there are at least one million home schooled children in the United States suggests that the home schooling alternative should not be dismissed lightly!

2. Math Enrichment Versus Reading Enrichment. How can you help your child continue to build on the gains made under the Plan once she begins attending elementary school? After all, the normal elementary school experience is a far cry from the type of pace and challenge that she has routinely encountered under the Plan!

Our answer to this question differs for the two principal content areas of the Plan, math and reading.

(a) Reading Enrichment. Obtaining appropriate reading enrichment is generally not much of a problem. Elementary school teachers are interested in reading; they teach it well, and have a real love for books. Thus, they are well-prepared to embrace and encourage a capable young reader. Also, it is relatively easy as a practical matter to provide individualized enrichment in the area of reading; most teachers can provide students with excellent books at all levels of difficulty, and the same processes that help students learn from the reading of a normally assigned book will usually work equally well with a harder book. In large part, the simplicity of enriching a child's reading experience within a diverse class flows from the fact that reading is not a sequential form of knowledge in the same fashion as math. Thus an enriched reading assignment can simply be a harder iteration of the same assignment that is being given to the class as a whole – e.g., a plot analysis, but of a harder, more complex book. For these reasons, appropriate enrichment of a Plan participant's reading experience generally does not require grade-level advancement, but instead can be done in the form of in-class enrichment. And, as a result, the key to helping your child have an appropriate and interesting elementary school reading experience turns out to be enlisting the teacher as an ally in the project, a topic that is discussed in section 3 below.

(b) Math Enrichment. On the other hand, it can be difficult for an elementary school teacher to provide adequate math enrichment for a student who is as advanced as the typical Plan participant. This is due in large part to the fact that mathematics is sequential in nature, with each new idea building on prior work. Thus, math enrichment requires a new curriculum, not just a new assignment! Also, although there are of course exceptions, the typical elementary school teacher is less comfortable with math than with reading, making in-class math enrichment more difficult.

Even larger-scale efforts at math enrichment by the elementary schools – e.g., district-wide enrichment programs for students who are identified as being gifted at math -- can fall short of the mark, for the same types of reasons. Teachers teach what they know, and all too often even the enrichment teachers do not really know math. So instead they teach brain teasers, or art disguised as math (e.g., tessellations), or – worst of all! – they teach rules of behavior, sort of a Robert's Rules of Order for young children.[2]

For example, school districts in Pittsburgh emphasize a program of math enrichment known as Calc-U-Solve. This program features problem-solving competitions among elementary school children, both individually and in teams. In principle, this is an excellent idea: the problems can on occasion be reasonably challenging and educational, and the children are happy and highly motivated by the competitive elements and also by the teamwork elements of the program. Unfortunately, the program is often administered by teachers who are "not math people," so it ends up failing to provide meaningful math enrichment. The emphasis is not upon math insight or correct math work; instead, it is upon procedural details – "rules of order." This leads on occasion to absurd results! A heavily emphasized "rule of order" in a recent competition was that a child who failed to write the correct units in her answer would receive no points. One of the questions in that competition was:

> "The book store had 28 mathematics books for sale. They recently sold half the math books and 3 books about the Civil War. How many math books are left?"

Some children answered "14," some answered "14 books," and some answered "14 math books." This presented the judges with a great, almost insuperable problem, one which was debated hotly for almost twenty minutes before a solution was reached. Clearly, the answer "14" was wrong (although all of the math work had been done correctly), because the answer did not include units. And clearly "14 math books" was correct; it was the perfect answer. But how about "14 books"? At first, the arbiter decided that this answer would be marked incorrect; later, this ruling was reversed and credit was given retroactively to "14 books" as well as to "14 math books."

Another "rule of order," added spontaneously after a few rounds of competition, was that children would not receive any credit at all unless they wrote their names on the backs of their papers – writing their names on the front would not suffice.

Do you see the point? Do you see the problem? The emphasis was in all the wrong places! The math problems were OK, but in truth were fairly easy for most of the fourth grade contestants and were generally trivial for a fourth grade Plan participant. The difficulty of the competition lay in the requirement that contestants adhere to a series of detailed procedural rules having very little to do with math. This was not a math competition; it was a competition in following behavior instructions.[3]

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2 Robert's Rules of Order, General Henry M. Robert (Chicago: Scott, Foreman, 1915) sets forth generally accepted rules of parliamentary procedure.

3 In order to lay to rest any possible concerns that these criticisms are motivated by some sort of sour grapes, we should point out that the Plan participant who was involved in this competition actually won at both the team and individual levels.

So, how should you approach the question of math enrichment in the schools for your child? Here are a few ideas:

(i) Assess the Teacher. Most importantly, assess your child's teacher. Is she interested in math? Does she seem to be good at math? Does she have a plan for enriching her students' math experience? Is she enthusiastic about that plan? If your child is fortunate enough to have a teacher who likes math, is good at math, and has a plan to provide meaningful math enrichment – and such teachers do exist! -- stick with that teacher! You have a gold mine!

(ii) Attempt to Have Your Child Accepted Into the School's Standard Enrichment Program. Usually these programs are more of a gesture than a real effort at enrichment, but they cannot hurt! And your child will feel good about herself if she does qualify.

(iii) Consider Advancement In Math Only. Our experience suggests that, absent an unusually good teacher and/or school system, regular math enrichment often will not suffice for a Plan participant. In such circumstances, you should consider whether your child should be accelerated into a higher math curriculum, matching her actual level of expertise. For example, a second grade Plan participant might fit well into a third or fourth grade math class. There may be small gaps in her knowledge, since the Plan does not attempt to mirror the elementary school math curriculum, but these gaps have not proven to be of any real significance in practice. (Of course, if the teacher is an obstacle rather than an ally, these gaps can be used as an excuse to deny acceleration.) The quid pro quo is that in the areas of fundamental math skills, which are emphasized in the Plan, the young Plan participant will most likely be more proficient even than her advanced-level classmates.

As a practical matter, advancement within the elementary school system generally will not have significant immediate educational benefits for your child. Although she will be at a higher level, the procedures and requirements for advancement make it unlikely that she will be able to advance to her actual level of achievement, so that even after advancement she most likely will not see very much in the way of new material. Also, if she continues to work under the Plan or a similar program, as recommended in section 4 below, her rapid pace will soon outstrip the work in her advanced class.

So, then, why advance?

There are two significant benefits of advancement: (a) it moves your child more quickly to higher grade levels, where math courses and teachers tend to be better; and (b) it boosts your child's sense of self-esteem. Both are significant, and they provide sufficient reason to seek advancement where appropriate.

" . . . we recommend advancement in math class only; for social reasons, we strongly discourage skipping a grade across the board."

Note that we recommend advancement in math class only; for social reasons, we strongly discourage skipping a grade across the board. Even if the child can handle the academic load in a higher grade (as is often the case for Plan participants), she may well pay a social price for skipping a grade. For example, by virtue of being a year younger than her classmates, she may have a harder time making a sports team in high school, and dating may be an issue, especially in that key year when her classmates are able to drive but she cannot.

Most schools will reflexively resist advancement. After all, the standard curriculum was designed by experts, bearded men who smoke pipes and have corduroy patches on the elbows of their jackets;[4] if the curriculum is good enough for most children, it is good enough for your child. Trust me, we Pittsburgh Planners have had a great deal of experience with this version of the famous "NIH Syndrome!"[5] For example:

" . . . the standard curriculum was designed by experts, bearded men who smoke pipes and have corduroy patches on the elbows of their jackets . . ."

• A friend of one Plan participant (not herself a participant) privately asked her math teacher to support her efforts to skip Integrated Mathematics 1 and go directly to Integrated Mathematics 2. The teacher reacted to this display of enthusiasm and zeal by announcing publicly to all of his classes that such advancement was <u>never</u> appropriate and that he would never support it, for any student under any circumstances.

• One Plan participant sought to self-study Pre-Calculus during the summer after her seventh grade year so that she could take Calculus early (in eighth grade). The head of the school district's math program tried to discourage this, then relented – very reluctantly – in the face of the student's offer to take the standard Pre-Calculus final exam. In the process of preparing for the exam, the student asked this administrator if the course covered all twelve chapters of the book. The administrator, who had taught the course herself, emphatically responded that the student was indeed responsible for all twelve chapters. In fact, the course has <u>never</u> covered more than <u>six</u> chapters![6] When the student did very well on the Pre-Calculus final, the administrator's assistant advised her, reluctantly and very apologetically, that although she had easily qualified for the AP Calculus course she would not be allowed to take it, but instead could take only a lesser Calculus course.

• Another Plan participant sought to take Integrated Mathematics 1 in fourth grade. Although he was already working two grade levels above his age group, and this would

4 And if that is not an expert, I do not what is!

5 "Not Invented Here Syndrome."

6 Although obstruction to advancement is common, it is rare that one encounters an outright lie like this!

merely have involved one additional year of advancement, the math coordinator refused to test him for this placement. Her reason? The child was in her view not emotionally mature enough to take Integrated Mathematics 1 with sixth graders. Her recommendation? That he instead simply move into her regular sixth grade math class, also consisting of sixth graders. When the logical inconsistency of this approach – avoiding placing a child with sixth graders by placing him with other sixth graders – was pointed out, the principal overrode the math coordinator and allowed the desired placement.[7]

• A local school district's program for advancement of third graders to fourth grade math (a one-year advancement) requires among other things that the third graders achieve a score on a fifth-grade math test (the math SCAT) that puts them at or above the 90th percentile of fifth-graders taking the test. In other words, to qualify for <u>fourth grade</u> math, a child must demonstrate excellent performance at <u>fifth grade</u> math! This is obviously not logical, but it is revealing. The school system does not really want to provide advancement as a regular option, because that would create administrative difficulties and suggest that the usual program is not adequate, particularly if it turned out that many children qualified for advancement. Instead, advancement is limited to a select few children who can be labeled "geniuses," i.e., people who are so extremely gifted that no one would expect them to be well-served by the school's usual program of math education. This program of advancement appears to be aimed not at providing children with the education they need, but instead at defusing parental pressure for better math education while preserving the status quo as much as possible. In other words, it is a token gesture, a preemptive move, an "overflow valve" that releases just enough parental pressure to allow the school system to proceed without real change in its math program.[8]

Be forewarned; this type of resistance is ubiquitous, and it may well be beyond your power to overcome. If you find that you are overwhelmed by the forces of inertia and mediocrity, do not panic, do not be alarmed! As noted above, the actual educational benefits of advancement are probably fairly minimal, particularly if you continue to work with your child under the Plan or a similar program (see discussion below). But, if you want to take a stab at advancement, you must enlist your child's teachers as your allies in order to bypass this type of resistance. Which leads us to our next section …

3. <u>Enlist Your Child's Teachers As Your Partners</u>. Perhaps the most important – and trickiest -- factor in your child's continued rapid progress will be the relationship you establish

7 …and the child did very well.

8 Almost every educator will agree that the American system of math education is badly broken, but almost no educator will admit that the problem exists in his or her district.

between that progress and your child's teachers.

Take a step back, and consider the picture your child's teacher will see when you approach her and tell her that your child needs special attention because she is very advanced and capable. Trust me, the teacher has heard this before! She will be polite; she will manufacture a polished, somewhat glassy smile, and she will give you one of the standard responses from the Educator's Handbook of Responses to Obnoxious Parent Comments[9] (e.g., "Thank you so much for that input! Suzie has been such a delight to have in class! She is a very special girl! And we will pay very special attention to making sure that she is challenged appropriately!").

Does this irritate you? Well, it should not! Because many parents say exactly the same thing about their children, and the teacher's long experience reveals that there is almost no relationship between a parent's view of a child's capabilities and the actual level of the child's capabilities! There is a difference between you and the other parents, because your child has put in years of effort working with you under the Plan. But the teacher cannot be expected to understand this, or to believe it on your say-so.

"Let me say this another way: as a parent, you have practically no credibility at all as a judge of your child's giftedness or special talents."

Let me say this another way: as a parent, you have practically no credibility at all as a judge of your child's giftedness or special talents. Your child's teacher has heard hundreds of parents explain why their child is special, and deep down, she is sick of it! And she discounts it about ninety-nine percent. [10]

So, how do you go about getting Suzie the accommodations and special treatment that she does in fact need, and has in fact earned through her years of effort? How do you overcome the almost-conclusive presumption of "parental jerkiness?"

You do three things.

First, have patience; Rome was not built in a day, and nothing will happen overnight no matter what you do. And – that is OK! Your child will be just fine working in the usual fashion on normal classroom material for a while, especially if you continue enrichment activities outside of the school.

If you feel that this is wrong, that your child will suffer from a lack of immediate challenge, please analyze your motives. Although you are very eager to have your child recognized as being exceptional, that eagerness is probably not motivated by a concern for your child's cognitive well-being; in fact, the odds are pretty good that your eagerness is an example of the very kind of parental jerkiness that teachers encounter all the time! You must fight that impatience, because it is probably

9 I do not know for sure that such a book exists, but I strongly suspect it.

10 Conversely, a parent who reports a child's problem area is taken very seriously.

a manifestation of ego rather than a true concern for your child's well-being.

Please do not be offended by this; every parent suffers from the same type of impatience that is being described here; that is part of being a parent. We cannot eliminate these impulses – they are probably baked into our being by thousands of years of evolution – but we can recognize them and transcend them. We will be asking the teacher to overcome her biases against pushy parents; the least we can do is reciprocate with a similar effort of our own!

Your child will be in school for many, many years. It is OK if she is not given special attention for a while. By being patient, you allow the other two approaches described below to come to fruition.

Second, after a decent period of time has elapsed, preferably in response to some sort of signal from the teacher that your child is performing well, mention that she loves school but sometimes asks at home for additional challenges. Lead this conversation into the idea that your child should be tested by the school for some sort of special program (e.g., a gifted program, acceleration in math, participation in accelerated reading sections, etc.). The goal is to <u>let the school discover on its own, through its own testing procedures or otherwise, that your child has unusual abilities and knowledge</u>. Teachers and school administrators believe their own test results! Often, the testing will occur automatically or without any prompting from you. If your child does not distinguish herself on the tests . . . . that is right; remain patient. Wait a while and then try again.

You may want to obtain a copy of the mathematics text being used by the grade above your child's grade level and teach your child any topics in that book that are gaps in her existing knowledge – once this has been done, you can offer (very effectively) to have your child take the final test from that course.

Third, once the school has begun to recognize your child's special situation, <u>enlist the teacher as your partner</u> in your child's success. What does this mean? First and foremost, it means <u>sharing the credit</u> for your child's performance. You may believe that it is almost entirely your efforts under the Plan that have led to your child's astonishing above-grade-level success at reading and math, but stifle that notion; put it under your hat and keep it there. You do not need the credit; your goal is altruistic and child based – you want your child to succeed. So give the credit to the teacher, liberally and without reservation. Say things such as, "Suzie has just blossomed in your class. She is truly enjoying the challenges you give her –- and she is advancing so rapidly! What are you doing?" If this is true, the praise and credit is deserved. If it is not, the praise and credit usually become self-fulfilling; the teacher will be highly motivated to "continue" (i.e., begin) a project that is going so well and for which she is receiving so much recognition.

By making the teacher your partner in your child's success, you do two things. First, you remove the teacher as a possible obstacle to your child's success. If your child is succeeding due to things outside the classroom, there is an implication that the classroom activities – i.e., the teacher's life and work – are somehow inferior or ineffective. The teacher is only human; she will not take this lying down. Instead of supporting your child, she will have a vested interest in showing that your child is not exceptional. Second, you gain the benefit of the teacher's efforts on behalf of your child. The teacher is almost certainly a better teacher than you, and she is in the profession for a reason – she loves to see children learn and grow, and revels in her contribution to their success. Whether or not the credit you give the teacher is initially deserved, it will be soon; she will become a great force for your child's continued progress.

A final note on sharing of credit – be sure to share it in a way that benefits the teacher. For example, send her a thank-you note and copy the principal, so that the note is included in her file. Nominate her for awards for teaching excellence. And give her the respect that her position and efforts deserve – in other words, listen to her! In truth, Plan or no Plan, she is the expert and you are the novice, and you are very fortunate to have her on the team.

Perhaps this three step approach of patience, school-initiated assessment, and sharing of credit seems a little Machiavellian. It is not intended to be so! Instead, it reflects a realization of the difficult background created by aggressive parents, and a respect for the school, the teacher, and their institutional processes. We consciously share credit even before it is deserved for two reasons: (i) we feel that it helps overcome the unfortunate background created by "parental jerkiness," and (ii) we believe that the sharing of credit will be fully deserved at the end of the day. In our experience, teachers and administrators have overwhelmingly ended up being great forces for good in the continued development of Plan participants.

4. Continue the Plan! In Chapters 1 – 5 we concluded that our society is simply not getting the job done in the areas of math and science education. Although we ultimately branched off into a preschool Plan aimed in part at "vaccinating' children against the cognitive perils of elementary school, we did not do anything to solve the fundamental problem: the schools remain inadequate in the areas of math and science education.

Given that fact, why not continue the Plan into the elementary school years? After all, the Plan ended where it did simply because it had reached a convenient breaking point, arbitrarily chosen to coincide with Sammy's entry into first grade. You will find that it is still quite easy, even in your child's elementary school years, to find time for regular fifteen-minute homework sessions. And you will also find that the vast cognitive benefits of the Plan continue unabated into those years.

If you and your child enjoyed participating in the Plan during these past few years, we urge you to continue to do so, either by using new (ever more advanced) homework sheets that you create yourself or by working out of above-grade textbooks or the books by Edward Zaccaro (or even, if it becomes available, the follow-up volume to this one, Newton Ascendant: The Pittsburgh Plan, Part 2), all as discussed at the beginning of this Chapter. As before, the continuation of the Plan is not intended to replace standard educational activities such as time in school or school homework, but instead serves as an important supplement, siphoning a small amount of time away from other activities such as television, video games, Celtic dance lessons, and the like.

Do older children enjoy continuing the Plan? Let me give you just one example of the types of things Plan participants learn to do in the next few years, well in advance of entering middle school. Plan participants can do the following problems in their heads, in a matter on two or three seconds (and not by rote memorization):

83 x 77 = 6391
28 x 32 = 896
39 x 41 = 1599
18 x 22 = 396
etc.

Sooner or later, a problem like this will come up in a classroom discussion. How do you think a child feels when she is able, casually, to state the answer? It is a very cool moment! And this is just one minor skill out of many that are learned in the continuation of the Plan. The following pages give examples of the type of work that is done under the continuation of the Plan[11] by a seven-year-old participant:

11 Although these "in-the-head" multiplications have the appearance of a parlor trick, they spring from an important math concept: for any two positive numbers a and b, $(a+b) \times (a-b) = a^2 - b^2$. Thus, for example, 83 x 77 can be thought of as (80+3) x (80-3), which from the above equation equals $80^2 - 3^2 = 6400 - 9 = 6391$. Q.E.D. ("Quite Exciting Demonstration!")

Name: Cody Date 4-21-98

V, A+ V.E O.W. 100%

(1) Cody can run 5 mph. She runs for 4 hours. How far does she run? 20 miles

hours

$5 \frac{miles}{\cancel{hour}} \cdot 4 \cancel{hour}$

$$\begin{array}{r} 5 \\ \times 4 \\ \hline 20 \end{array}$$

How far can she run in $\frac{1}{2}$ hour? $2 \frac{1}{2}$ miles

60 minutes per hour

$\frac{10}{3} \; 3\frac{2}{3} \times 5 = 3\frac{1}{3}$ $\frac{2}{3}$ hrs $\cdot$ 5 miles/hour

How far in 40 minutes? $3\frac{1}{3}$ miles ✓

How far in $2\frac{1}{2}$ hours? $12\frac{1}{2}$ miles ✓

$2\frac{1}{2}$ hrs $\times 5 \frac{miles}{hour} = 12\frac{1}{2}$ miles

How long will it take her to run 20 miles? 4 hours

$\frac{20 \text{ miles}}{5 \frac{\text{miles}}{\text{hour}}}$

23 miles? 4 hours 36 minutes

$\frac{23}{5}$ mi $\cdot$ $\frac{hour}{\cancel{miles}}$

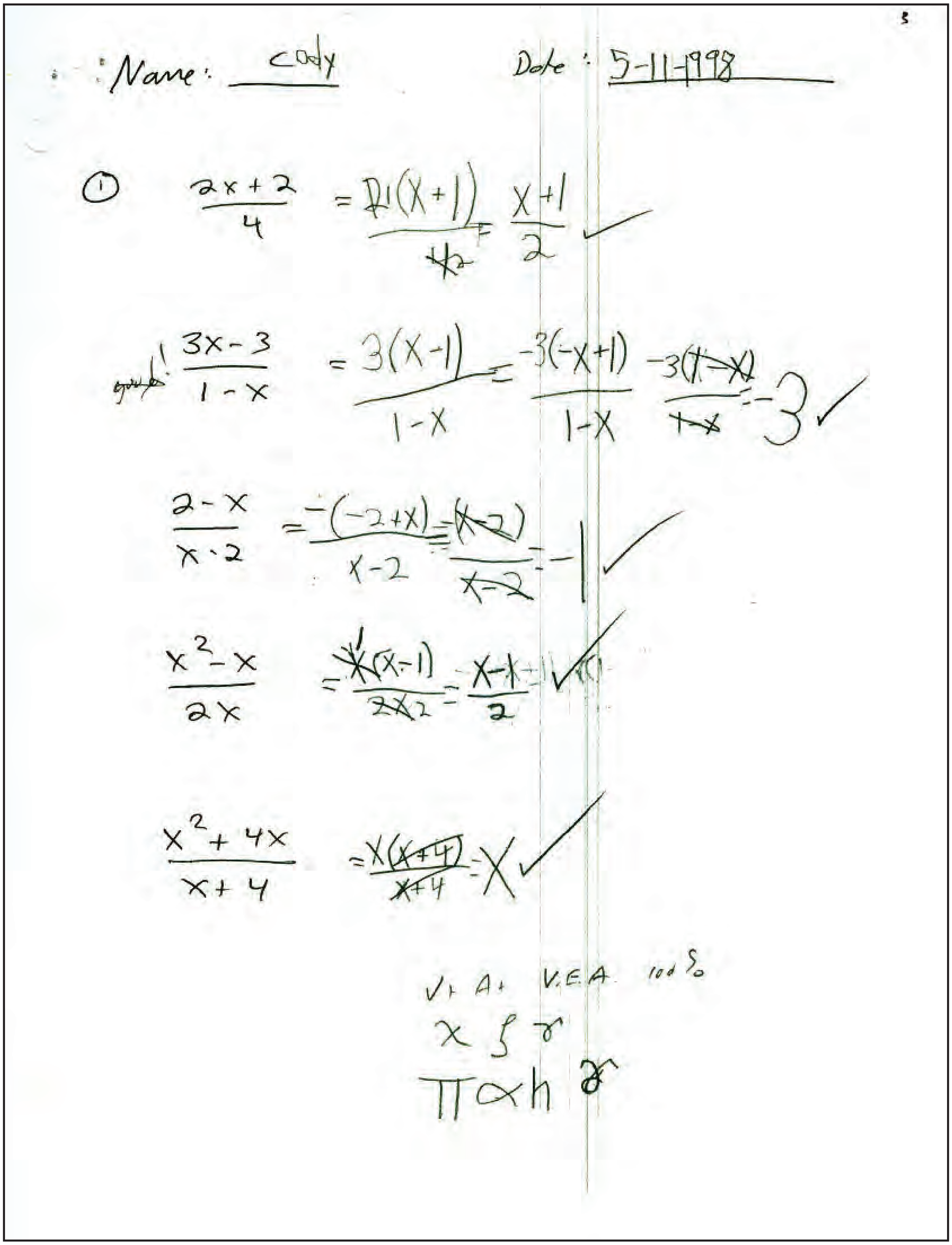
$\frac{180}{36}$

$\frac{3}{5} \cdot 60$

8 miles? 1 hour and 36 minutes ✓ 23 miles

$\frac{8 \text{ miles}}{5 \text{ miles}}$ hour

$5 \frac{}{h}$ $\frac{23 \text{ miles}}{5} \cdot \frac{hour}{\cancel{mile}}$



Name: cody Date: 5-11-1998

① $\frac{2x+2}{4} = \frac{\cancel{2}1(x+1)}{\cancel{4}2} = \frac{x+1}{2}$ ✓

good! $\frac{3x-3}{1-x} = \frac{3(x-1)}{1-x} = \frac{-3(-x+1)}{1-x} = \frac{-3\cancel{(1-x)}}{\cancel{1-x}} = -3$ ✓

$\frac{2-x}{x-2} = \frac{-(-2+x)}{x-2} = \frac{-\cancel{(x-2)}}{\cancel{x-2}} = -1$ ✓

$\frac{x^2-x}{2x} = \frac{\cancel{x}(x-1)}{2\cancel{x}} = \frac{x-1}{2}$ ✓

$\frac{x^2+4x}{x+4} = \frac{x\cancel{(x+4)}}{\cancel{x+4}} = x$ ✓

V+ A+ V.E.A 100%

$x\ \xi\ \gamma$

$\pi\ \alpha\ h\ \partial$

4

(2) $(x+2)^2 = (x+2)(x+2) = x^2+2x+2x+4 = x^2+4x+4 = (x+2)^2$ ✓

$(x-1)^2 = (x-1)(x-1) = x^2-x-x-1 = x^2-2x-1 = (x-1)^2$ ✓

$(x+1)(x-1) = x^2-x+x-1 = x^2-1 = (x+1)(x-1)$ ✓

$(x+2)(x-2) = x^2-2x+2x-4 = x^2-4$ ✓

$(x+a)(x-a) = x^2-x\cdot a+a\cdot x-a^2 = x^2-a^2$ ✓

(3) $\cancel{5}\,\frac{4}{\cancel{5}}x = 8$

$4x = 40$

$\frac{40}{4} = 10$

$x = 10$ ✓

$\cancel{3}\,\frac{2}{\cancel{3}}x = 12$

$2x = 36$

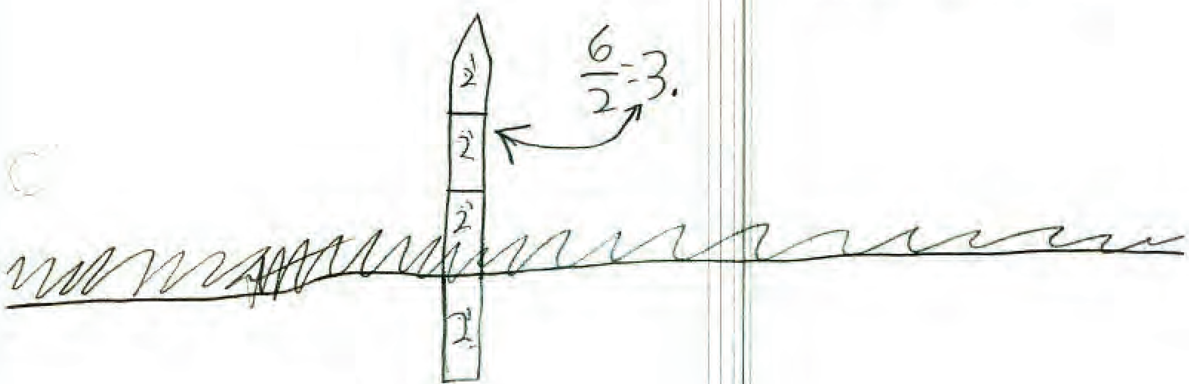
$\frac{36}{2} = 18$

$x = 18$ ✓

5

(4) A fencepost will only stand up if at least $\frac{1}{4}$ of it is buried. If a fencepost is 6' above the ground, it must be at least <u>8</u>' long total.

TOTAL L = X



$$\cancel{4}\,\frac{3}{\cancel{4}}x = 6 \cdot 4$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{24}{3} = 8$$

$$x = 8$$

Name: cody Date: 5-12-98

① $2x + \cancel{1} = 6$

$-1 \quad -1$

$\cancel{2}x = \frac{5}{2} = 2\frac{1}{2}$

$x = 2\frac{1}{2}$ ✓

$6x - 4 = 3x + 7$

$+4 \qquad +4$

$6x = 3x + 11$

$-3x \quad -3x$

$\cancel{3}x = \frac{11}{3} = 3\frac{2}{3}$ ✓

② Square roots: Example: $\sqrt{9} = 3$, since $3 \cdot 3 = 9$

$\sqrt{25} = 5$ ✓

$\sqrt{36} = 6$ ✓

$\sqrt{4} = 2$ ✓

$\sqrt{16} = 4$ ✓

✓+ A+ V.E.S.e.

100%

$\equiv \xi \xi$

$\psi \pi \lambda$

(3). What happens if you square a square root?

$\overset{3}{\sqrt{9}} \cdot \overset{3}{\sqrt{9}} = (\sqrt{9})^2 = \underline{\sqrt{81} = 9}$

$(\sqrt{4})^2 = \underline{\sqrt{16} = 4}$

$(\sqrt{25})^2 = \underline{\sqrt{5}=25}$

$(\sqrt{2.693})^2 = 2.693$

$(\sqrt{3})^2 = \underline{\sqrt{9} = 3}$

$(\sqrt{7})^2 = \underline{\sqrt{49} = 7}$

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$

(4) Most square roots cannot be simplified. For example, $\sqrt{7}$ cannot be simplified. ($\sqrt{4}$, $\sqrt{9}$, etc. are exceptions).

Put check marks next to the square roots that cannot be simplified:

$\sqrt{16}$

$\sqrt{11}$ ✓

$\sqrt{2}$ ✓ $\sqrt{4}$ ___

$\sqrt{1}$ = ✓ = 1 $\sqrt{5}$ ✓

$\sqrt{3}$ ✓ $\sqrt{6}$ ✓

1.732....

5. Adding square roots. First, simplify if you can! Then, add or subtract square roots that are the same. (Square roots are like different variables -- if they are different (x, y for example), they can't be added or subtracted).

1
4
9
16
25
36

Example $\sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$ -- can add

$\sqrt{2} + 2\sqrt{3} = \sqrt{2} + 2\sqrt{3}$ -- can't add

$4\sqrt{3} + \sqrt{2} = 4\sqrt{3} + \sqrt{2}$ ✓

$x + 2x$
$1x + 2x$

$2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$ ✓

$\sqrt{5} - \sqrt{5} =$ [illegible]

$x - x = 0$

chdl − chdl = 0

cof − cof = 0

$2 - 2 = 0$

$4\sqrt{2} - 5\sqrt{2} = -1\sqrt{2} = -\sqrt{2}$

$\sqrt{7} + \sqrt{14} =$ [illegible] $= \sqrt{7} + \sqrt{14}$

Thank you for trusting us to help you with your child's early education!
We hope that you and she have enjoyed the Plan.